The Relationship Between Teachers' Mathematical Content and Pedagogical Knowledge, Teachers' Perceptions, and Student Achievement

Author(s): Patricia F. Campbell, Masako Nishio, Toni M. Smith, Lawrence M. Clark, Darcy L. Conant, Amber H. Rust, Jill Neumayer DePiper, Toya Jones Frank, Matthew J. Griffin and Youyoung Choi

Source: *Journal for Research in Mathematics Education*, Vol. 45, No. 4 (July 2014), pp. 419-459
Published by: National Council of Teachers of Mathematics
Stable URL: http://www.jstor.org/stable/10.5951/jresematheduc.45.4.0419
Accessed: 02/03/2015 19:20

Journal for Research in Mathematics Education
2014, Vol. 45, No. 4, 419–459

The Relationship Between Teachers' Mathematical Content and Pedagogical Knowledge, Teachers' Perceptions, and Student Achievement

Patricia F. Campbell
University of Maryland

Amber H. Rust
Anne Arundel Community College

Masako Nishio
University of Maryland

Jill Neumayer DePiper
Education Development Center

Toni M. Smith
American Institutes for Research

Toya Jones Frank
George Mason University

Lawrence M. Clark
University of Maryland

Matthew J. Griffin
University of Maryland

Darcy L. Conant
Notre Dame of Maryland University

Youyoung Choi
Hanyang Cyber University

This study of early-career teachers identified a significant relationship between upper-elementary teachers' mathematical content knowledge and their students' mathematics achievement, after controlling for student- and teacher-level characteristics. Further, the mathematical content and pedagogical knowledge of middle-grades teachers were each directly and positively related to their students' mathematics achievement, with and without teacher-level controls. Significant interactions emerged between teachers' perceptions and knowledge influencing student achievement. Teachers' claimed awareness of their students' dispositions toward mathematics interacted with upper-elementary teachers' content knowledge; middle-grades teachers' beliefs regarding modeling mathematical solutions and organizing instruction to support incremental mastery of skills interacted with both content and pedagogical knowledge. Findings provide evidence of the relevance of teacher knowledge and perceptions for teacher preparation and professional development programs.

Key words: Student achievement; Teacher beliefs; Teacher knowledge

---

In accordance with *JRME* policy regarding potential conflicts of interest with the editor, the review process for this manuscript was handled by Guest Editor Barbara J. Dougherty.

This material is based upon work supported by the National Science Foundation under Grant No. DRL 0426253. Any opinions, findings, and conclusion or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

There is a logical premise that teacher knowledge "directly and positively affects classroom practice" and subsequently student achievement (Smith & Esch, 2012, p. 2). But with the exception of studies of high school mathematics teachers (Goldhaber & Brewer, 2000; Monk & King, 1994; Rice, 2003), empirical evidence is inconsistent, perhaps because of reliance on proxy measures for teacher knowledge, such as completion of a degree or coursework (Wayne & Youngs, 2003; Wilson, Floden, & Ferrini-Mundy, 2002).

Recognizing this, as well as Shulman's (1986) theoretical proposal that teacher knowledge encompasses and bridges content-specific knowledge with knowledge of and applied in the practice of teaching, the National Mathematics Advisory Panel (2008) issued a call for researchers to define "more precise measures" of both mathematical and pedagogical knowledge (p. 38) in order to reveal potential relationships between teacher knowledge and student learning.

At the same time, recent studies examining mathematics instruction noted that teachers seem to draw not only on their pedagogical and content knowledge when teaching but also on their beliefs (Beswick, 2007; Bray, 2011; Gellert, 2000) and their awareness of classroom conditions and interactions (Sherin, Jacobs, & Philipp, 2011). Indeed, current conceptualizations suggest that teachers' knowledge, pedagogical repertoire, beliefs, and interpretations of classroom interactions may interrelate in a dynamic, interconnected system that influences a wide range of instructional practices and approaches (Philipp, 2007).

Mathematics Teacher Knowledge and Student Achievement

Researchers at the University of Michigan initiated the study of mathematical knowledge for teaching. They investigated "recurrent tasks and problems of teaching mathematics" as well as the "mathematical knowledge, skills and sensibilities" displayed by the teachers managing those tasks (Ball, Thames, & Phelps, 2008, p. 395). Then, interpreting that analysis, they developed a multiple-choice instrument that directly measured elementary school teachers' knowledge of aspects of common and specialized content knowledge for teaching mathematics (Hill, Schilling, & Ball, 2004). These researchers identified a significant relationship, as measured by this instrument, between teacher content knowledge and student mathematics gain scores on standardized achievement assessments in first and third grade (Hill, Rowan, & Ball, 2005).

Ball, Thames, and Phelps (2008) proposed categories of mathematical knowledge for teaching that span Shulman's (1986) *subject matter knowledge* and *pedagogical content knowledge*. Although their measure of teacher knowledge is a recognized achievement in mathematics education research, the analysis reported in their study investigating the relationship between teacher knowledge and student achievement did not differentiate pedagogical content knowledge from mathematical content knowledge (Hill et al., 2005).

In an investigation of teacher knowledge involving experienced Grade 10 mathematics teachers in Germany, Baumert et al. (2010) empirically distinguished teachers' knowledge of mathematics content and pedagogy in an open-ended assessment. These researchers identified a statistically significant relationship between student achievement and teachers' mathematical content knowledge as well as between student achievement and teachers' pedagogical content knowledge for mathematics. Because teachers with stronger pedagogical content knowledge assessed their students' understanding at the end of instructional units with tasks requiring a higher cognitive demand and because surveyed students of these teachers reported higher levels of "instructional quality," Baumert et al. identified pedagogical content knowledge as the stronger predictor of student learning, presuming teachers' adequate content knowledge. Although the statistical analysis in the Baumert et al. study used multilevel models to control for selective tracking assignments at the student level, implications from this work for the U.S. educational system are cautionary because both teacher preparation and schooling programs in Germany are tracked (differing academic and nonacademic curriculum expectations for students with differing teacher-certification standards), resulting in systemic educational disparities.

Focus of This Study

The measures of teachers' knowledge in the Hill, Rowan, and Ball (2005) and the Baumert et al. (2010) studies were not designed to align with standardized measures of student achievement, nor did their study designs encompass teachers' beliefs. In contrast, in this study we investigated whether there is a relationship between student achievement and teachers' perceptions, by which we mean teachers' beliefs regarding mathematics teaching and learning and teachers' awareness of their students' mathematical dispositions. In addition, we aligned measures of teachers' mathematical content and pedagogical knowledge with expectations for student achievement, as expressed in state mathematics curriculum standards and measured in state assessments. By aligning its measure of teacher knowledge with the content upon which students are assessed, this study may inform both teacher education and education policy efforts to strengthen and support teacher quality that advances student achievement. Further, this investigation targeted mathematics teachers across Grades 4 through 8, grade levels distinct from those in the Hill et al. (2005) and the Baumert et al. (2010) studies. The research question addressed in this study was: *What is the relationship between teachers' mathematical and pedagogical knowledge, teachers' perceptions, and their students' achievement, controlling for student demographics, teaching experience, and teaching assignment?*

Conceptual Model

When teaching, mathematics teachers access their usable knowledge of mathematics content as well as their knowledge of mathematics teaching and learning. In addition, instructional decisions require teachers to weigh choices associated with many differing, and possibly incompatible, beliefs (Aguirre & Speer, 1999). Thus, instructional practices affecting student mathematics achievement may be influenced not only by teachers' knowledge and professional background but also by teachers' beliefs (Ball, 1991; Hill et al., 2008). Researchers (e.g., Murphy, Delli, & Edwards, 2004; Pajares, 1992; Philipp, 2007) have drawn a clear distinction between beliefs and knowledge, noting that, unlike knowledge, beliefs are evaluative and not verifiable or fact-based. While attempts to establish a connection between student achievement and teachers' professional background have not identified substantive predictability (Wilson et al., 2002), some researchers have suggested that teachers' awareness of their students' prior mathematical experiences and dispositions may allow classroom teachers to better meet the needs of students as learners (Boaler & Greeno, 2000; Martin, 2000).

Wilkins (2008) proposed a theoretical model relating teachers' content knowledge, attitudes, beliefs, and instructional practice. Figure 1 presents a modification of Wilkins' model, which incorporates professional development, teaching context, and student experiences and achievement. This model identifies teacher professional background and experience, knowledge, beliefs, and awareness variables that may influence instructional practice, and it acknowledges the mediating effect of teaching contexts and access to professional development. At the same time, there is emerging recognition that the experiences and dispositions of students may affect not only their achievement (Kilpatrick, Swafford, & Findell 2001) but also the beliefs of their teachers (Sztajn, 2003). Although direct measures of students' experiences and dispositions are not typically accessible, Figure 1 does identify related measurable variables characterizing students. However, other elements not referenced in Figure 1 do influence instructional practice. These include elements such as how teachers interact with and manage students, teachers' professional identity, the quality of available resources, the intended curriculum, and contextual factors in the classroom, school, and district.

In this study, we investigated only some aspects of the conceptual model depicted in Figure 1. In particular, we did not directly measure instructional practice. However, teacher knowledge and beliefs do potentially interrelate and influence instructional practice, which in turn influences student achievement. This report is limited to a broad exploration of the extent to which student achievement in mathematics is related not only to teachers' mathematical content and pedagogical knowledge but also to teachers' beliefs about mathematics teaching and learning and teachers' perceived awareness of their students' dispositions toward learning mathematics. A literature review located one refereed publication in a scholarly journal investigating the relationship between teachers' beliefs about teaching, student needs, and mathematics achievement (Love & Kruger, 2005). Although Love and Kruger (2005) noted a statistically significant correlation between student achievement and both a belief in student strengths and a willingness to "allow students to teach the class" (p. 95), that study did not simultaneously address teacher knowledge.

Teaching Experience

Teaching experience may be associated with teacher knowledge and may affect student achievement, but it is not evenly distributed across the population of

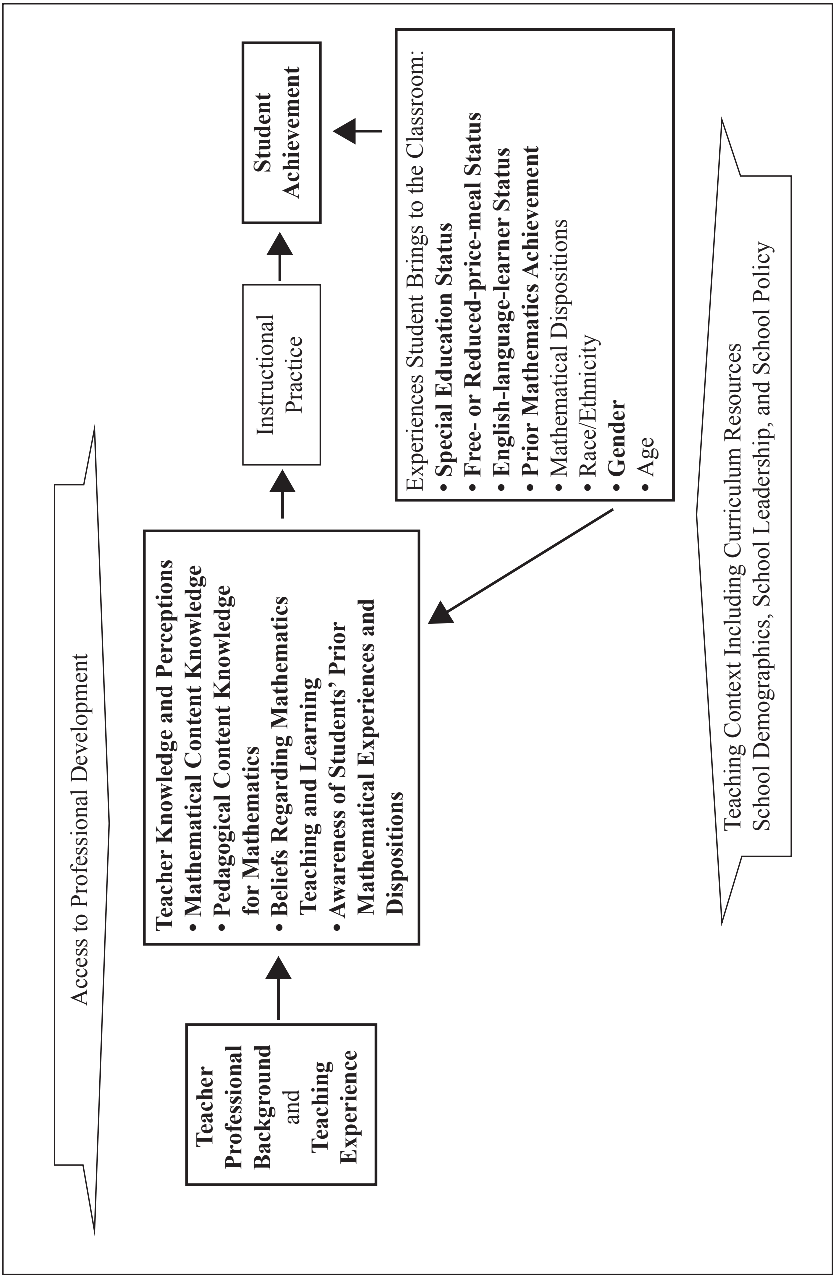

Figure 1. Conceptual model positioning teachers' knowledge and perceptions with teachers' professional experiences, instructional practice and student achievement, within the setting of teaching context and professional development. In this study we examined only the components shown in bold.

practicing teachers (Nye, Konstantopoulos, & Hedges, 2004; Rivkin, Hanushek, & Kain, 2005). In addition, attrition in the teaching force is less prevalent among more veteran teachers because many teachers who leave the profession do so within approximately their first 5 years of teaching (Darling-Hammond & Sykes, 2003; Ingersoll, 2001; Scott, Milem, Stuessy, Blount, & Bentz, 2006). Thus, from a policy perspective, an investigation of the relationship between student achievement and the knowledge and perceptions of less experienced teachers is useful. Investigations involving only early-career teachers may yield information informing the design of professional development and of on-site support that addresses their needs, thereby increasing retention. For these reasons, we limited the sample to teachers who had 6 or fewer years of teaching experience.

Teacher Knowledge: Definitions and Framework Development

Teacher knowledge frameworks frequently specify broad inventories for teacher education programs (e.g., Conference Board of the Mathematical Sciences [CBMS], 2001). In order to investigate whether teachers' mathematical content and pedagogical content knowledge relate to student achievement in unique ways, we focused on teacher knowledge that might most directly influence student achievement in mathematics as assessed on high-stakes, standardized state tests. Access to teacher and student achievement data was limited to the mid-Atlantic states of Delaware, Maryland, and Pennsylvania, thus defining a convenience sample. The mathematics content standards in these three states were similar, permitting identification of a multistate teacher and student sample and the development of a single teacher-knowledge measure suitable for administration to teachers in all three states.

We specified teacher knowledge by intersecting an analysis of teacher-knowledge and professional-licensure frameworks (e.g., Educational Testing Service, 2008; National Council for Accreditation of Teacher Education [NCATE]/National Council of Teachers of Mathematics [NCTM], 2003), including content identified by mathematicians and mathematics educators (Bush et al., 2005; CBMS, 2001; NCTM, 1991), with the Delaware, Maryland, and Pennsylvania curriculum standards for student mathematics (Grades 4–8). The measures of teacher mathematical content and pedagogical knowledge in this study were designed to address understanding associated with teaching the Grades 4–8 school mathematics upon which students are assessed (Delaware, Maryland, and Pennsylvania standards) and the understanding a teacher may draw on to teach that content. As such, no claim is made that this study's teacher-knowledge frameworks or its measures are exhaustive.

An underlying assumption in this study is that mathematical and pedagogical content knowledge are distinct yet possibly linked (Ball, Lubienski, & Mewborn, 2001). In order to distinguish pedagogical content knowledge and mathematical content knowledge empirically and to categorize teacher-knowledge assessment items accordingly, definitions of mathematical content knowledge and pedagogical content knowledge were established.

Mathematical content knowledge. In this study, we defined *mathematical content knowledge* (CK) as knowledge related to or underlying the school mathematics content assessed at Grades 4–8. This content could also be taught in later grades. For example, the mathematics referenced in the items assessing the content knowledge of upper-elementary teachers from Grades 4 and 5 could be held by or taught to secondary school students. CK comprises knowledge of mathematical facts and procedures as well as knowledge of mathematical concepts and generalizations. It includes what has been termed *common content knowledge* and *specialized content knowledge* for mathematics (Ball et al., 2008).

To focus the range of teachers' CK being assessed, we specified a mathematics content framework that identified the scope of items for teachers of students in Grade 4 or 5 and a different framework defining the scope of items for teachers of students across Grades 6, 7, and 8. Figure 2 depicts the procedure that was implemented to produce these two frameworks.

The mapping of identified teacher-content-knowledge indicators against assessed student mathematics objectives in Delaware, Maryland, and Pennsylvania produced two distinct matrices, with the listing of mathematics expectations for either the upper-elementary (Grades 4 and 5) or the middle-grades (Grades 6, 7, and 8) teachers defining the row variables in a matrix and the column variables identifying the three states. As depicted in Table 1, the values in the cells formed by the intersections of row and column variables specify the number of student mathematics objectives in a state's curriculum standards that were associated with the mathematics content expectation for teachers as listed in that row. Only those objectives upon which students were assessed were tallied. For a given teacher-knowledge expectation, if no associated assessed student objective was listed in the curriculum standards of a state, then Table 1 lists a zero in that cell.

Consider the meaning of selected rows from Table 1. During the review of teacher knowledge and professional licensure frameworks, one or more identified resources indicated that upper-elementary teachers should be able to represent, compare, order, or equate integers. However, as determined by its listing of assessed student objectives, only Pennsylvania assessed its fourth- or fifth-grade students' knowledge of some aspect of this content within its high-stakes state assessments. In contrast, the expectation that teachers should understand how a change in the measure of one attribute of an object or entity related to the change in the measure of another attribute of that figure or object was related to student assessment indicators in all three states.

The final step of the procedure identified those teacher-content-knowledge expectations, the row variables in the matrices, which had entries in each of the three columns specifying state indicators. Each of these listings, one for the upper-elementary teachers and one for the middle-grades teachers, defined the respective frameworks for teachers' CK as identified for this study.

Pedagogical content knowledge. In this study, we defined *pedagogical content knowledge* (PCK) for mathematics as knowledge of mathematics teaching and

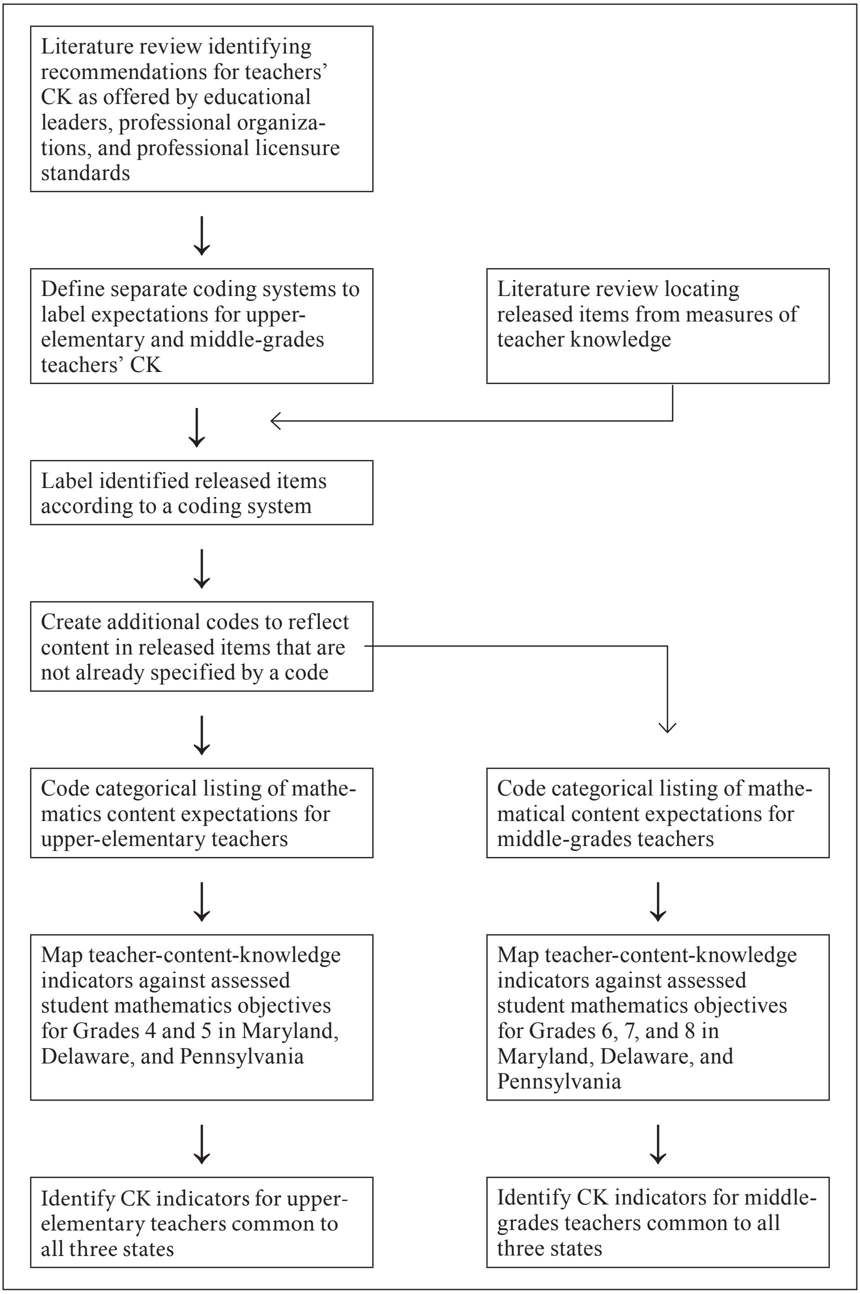

Figure 2. Process that yielded mathematics content frameworks specifying the focus and range for assessed CK of upper-elementary and of middle-grades teachers.

Table 1
Selected Upper-elementary Teacher Expectations Crossed with Assessed Student Objectives

Upper-elementary teacher content knowledge expectations	Number of assessed student mathematics objectives by state standards		
	Delaware	Maryland	Pennsylvania
Number and Operations: Rational Number or Integer Concepts or Representations			
Represent, compare, order, or equate integers	0	0	1
Represent, compare, order, or equate within or among decimals	4	4	2
Number and Operations: Number Theory or Number Systems			
Identify or apply real number properties (including closure) when solving problems involving any subset of the positive rational numbers	0	0	0
Determine or represent prime factorization (including exponents)	0	1	2
Geometry: Two-dimensional Geometry			
Express the hierarchy of quadrilaterals	1	1	0
Measurement			
Describe or determine how a change in the measurement of one attribute relates to the measurement of another attribute (e.g., area/perimeter; volume/surface area; angle/ray; area/circumference/radius)	2	3	1
Data Analysis			
Characterize, distinguish between, determine, or use measures of central tendency (mean, median, or mode) to answer questions about data sets, to solve problems, or to offer conclusions	1	3	1
Patterns, Functions and Algebra: Algebraic Concepts or Applications			
Identify, distinguish, or define: variable, expression, equation, term, inequality, polynomial, simplify, evaluate, or solve	2	0	0

learning that teachers might draw on or use in instructional practice when teaching the mathematics content assessed on high-stakes state assessments, but not knowledge that is typically taught to more advanced precollege students. Teacher PCK as assessed in this study comprised knowledge of students' understanding of or thinking about mathematics, knowledge of trajectories for teaching key mathematical topics, knowledge of emergent interpretations of mathematics in student work, and knowledge of how to respond to students' interpretations of mathematical content. This included what has been termed *knowledge of content and students, knowledge of content and teaching,* and *knowledge of content and curriculum* (Ball et al., 2008).

In mathematics education, pedagogical content knowledge can be characterized by domains (Ball et al., 2001). Our review of the literature from educational leaders, professional organizations, and professional licensure standards (e.g., NCATE/NCTM, 2003; NCTM, 1991) yielded four domains that comprised the PCK framework for this study: common student errors and misconceptions (Domain 1), mathematical representations and contexts (Domain 2), sense of order for mathematical content (Domain 3), and addressing and understanding students' interpretations of mathematics (Domain 4).

Teacher Perceptions: Beliefs and Awareness

Researchers addressing the beliefs of mathematics teachers have examined teachers' beliefs about the nature and structure of mathematics (Gellert, 2000; Szydlik, Szydlik, & Benson, 2003), the efficacy of particular pedagogical approaches (Peterson, Fennema, Carpenter, & Loef, 1989; Stipek, Givvin, Salmon, & MacGyvers, 2001), the role of the teacher and the role of the student in the mathematics classroom (Cooney, Shealy, & Arvold, 1998), perspectives on students' learning mathematics (Philipp et al., 2007), beliefs about students' abilities and capabilities (Goddard, Hoy, & Hoy, 2000), and the extent to which teachers were aligned with particular theories of instructional practices (Ross, McDougall, Hogaboam-Gray, & LeSage, 2003) or of learning (Beswick, 2007). To examine the relationship between teachers' beliefs and student achievement, we sought to characterize how teachers' beliefs about mathematics teaching and learning clustered across or at the intersections of belief categories by addressing the focus of mathematics classroom instruction, how instruction should be ordered and how mathematics classrooms and materials should be organized, how students learn mathematics best, and the roles of students and the roles of teachers in the mathematics classroom.

Teachers may hold similar beliefs about mathematics teaching and learning and possess similar levels of mathematical knowledge; yet they may hold differing interpretations of classroom interactions, the capabilities of their students, or what motivates their students to engage or disengage in the mathematics classroom. Although educational psychologists have considered the role of students' dispositions for some time, only recently have mathematics educators begun to address the critical role of students' mathematical dispositions (Kilpatrick et al., 2001;

Martin, 2000) and to examine pedagogical practices that encourage students to see themselves as learners who "think, negotiate, and understand" mathematics (Boaler & Greeno, 2000, p. 190). Researchers have suggested that teacher awareness and understanding of students' prior mathematical experiences and dispositions may be related to how teachers meet the learning needs of their students (Boaler & Greeno, 2000; Martin, 2000). Thus, in addition to measuring teachers' beliefs about the teaching and learning of mathematics, we sought to measure the extent to which teachers claimed to (a) have an awareness of their students' dispositions and (b) determine the nature of their students' dispositions. We measured these constructs along three dimensions: their students' perspectives regarding the importance of mathematics, their students' motivation for engaging in mathematical tasks, and their students' self-perceptions of mathematical ability. The viability of inclusion of these constructs has recently been supported by identification of teachers' awareness of their students' mathematical dispositions as a variable in assessments of mathematics teacher effectiveness and quality (Stanford Center for Assessment, Learning, and Equity, 2012).

Method

Subjects

The research team sought the cooperation of school districts in Delaware, Maryland, and Pennsylvania that would provide the study with anonymous individual student achievement and demographic data linked to individual teachers. Because data characterizing teachers' knowledge and perceptions could only come from direct measures, the research team sought the participation of teachers within those districts, once cooperating districts were identified.

As a first step, school districts across the three states were characterized in terms of student enrollment, distribution of student race or ethnicity, proportion of students receiving free and reduced-price meals, and geographic locale. Then an in-state sorting of districts was completed to yield separate listings, each referencing comparable school districts. A proposal requesting school district cooperation in this research effort was subsequently submitted to at least one school district in each listing, attending to geographic locale within a state when selecting these initial contact school districts in a listing. If a district did not agree to participate, another district in that listing was contacted until a sample of school districts crossing socioeconomic and geographic strata in each of the three states was acquired. Proposals were submitted to 47 districts, and 23 of these districts agreed to participate.

Although some districts had unique reasons for not participating in this research, the most common reasons were (a) a determination that study findings would not provide sufficient benefit or be of interest to the district; (b) a concern that accessing teacher-linked student achievement scores would be too labor intensive or impossible; and (c) an evaluation that the district was already sufficiently committed to research projects previously approved. An unexpected difficulty was that a number of school districts could not digitally link their database containing student achievement data from the state assessment with their student attendance or teacher schedule databases, making it impossible for these districts to sort electronically their student achievement files by teacher identifiers. We recognize the professional commitment and cooperation of the cooperating districts, because compiling the teacher-linked student data for this study was not a trivial endeavor.

The subjects of this study were the 266 upper-elementary and 193 middle-grades early-career teachers of mathematics who volunteered to participate in the study. Teachers were solicited through a project-designed electronic flyer distributed by cooperating school districts. Teachers interested in participating in the study registered through a secure website identified on the flyer. The registration process involved responding to questions designed to establish that the registrants were in their first 6 years of teaching, were employed by a cooperating district, and were responsible for teaching mathematics to students in Grades 4–8. A maximum number of teacher participants was set for each district in order to maintain a state-level demographic distribution over districts. If more teachers in a district applied than were needed, the applicants were categorized by their grade band (Grades 4–5 or 6–8) and then, within grade band, by their number of years of teaching experience. We then employed a first-applied, first-selected criterion within each grade band/experience category. Although the process of selecting school districts manifested an effort to secure representative demographic variance, the final determination of school districts and of teachers in this study was not random.

Nonrandom identification may introduce unmeasured factors that could influence teacher quality and student achievement. For example, teachers may seek or transfer to positions in schools similar to those that they attended or those enrolling students with social backgrounds similar to their own (Boyd, Lankford, Loeb, & Wyckoff, 2005). Although the declining national economic condition in the United States affected each of the communities served by the cooperating school districts during the period of data collection, differential migration among the early-career teachers in this study may have been minimized because none of the cooperating school districts were experiencing either high teacher turnover or persistent teaching vacancies.

An additional concern at the time of study conceptualization was that only those teachers who enjoyed mathematics or who were confident in their mathematical ability would volunteer to complete an assessment addressing mathematics content and pedagogy. To encourage participation, teachers were paid $350 for completion of tests of teacher knowledge as well as surveys of teachers' perceptions (beliefs and awareness), professional background, and instructional context. The 120 teacher-knowledge items were arranged within five test booklets, each containing 24 items. The test booklets and surveys were administered in an alternating fashion during a single nonschool day at a local nonschool site. The stipend was set to act as an incentive, attracting teachers with varying levels of mathematical confidence. A few school districts, however, had to decline to participate because they had a policy prohibiting cooperation with any effort that paid teachers a stipend. Although the analysis that follows applied statistical models addressing not only student demographics but also teacher experience and assignment, the limitation associated with nonrandom selection of teachers is acknowledged.

The cooperating school districts were unable to locate student data for 16 teachers (7 upper-elementary and 9 middle-grades teachers). Thus in this study we examined the teacher knowledge and perception data of 259 upper-elementary and 184 middle-grades teachers from 23 school districts across three states. The demographics, prior professional experience, and backgrounds of these early-career teachers are noted in Table 2 along with some additional characterization of their teaching assignment and school district location.

Data Sources

Student data. Table 3 presents the demographics of the students to whom these cooperating teachers taught mathematics in 2008–2009 and for whom their school district forwarded state mathematics achievement scores. Only data associated with students who completed their state's regular high-stakes measure as required under the No Child Left Behind Act of 2001 are included. Students who completed an alternate assessment as administered within Delaware, Maryland, or Pennsylvania in accordance with federal regulations are not represented. In those cases where more than one teacher taught a student, researchers communicated directly with school-district personnel and with teachers to clarify primary responsibility for mathematics instruction. These communications maintained student anonymity through use of numerically coded identifiers.

One of the school districts did not maintain records of individual student data regarding free and reduced-price meals. This district determined a poverty indicator at the school level based on U.S. census data and provided meals to all enrolled students at qualifying schools. Thus, the term *poverty indicator* is used in this investigation to represent the best available data that school districts could provide regarding the economic status of their enrolled students.

Teacher-knowledge assessments. The process depicted in Figure 2 yielded frameworks specifying mathematical teacher-knowledge topics at the upper-elementary and middle-grades levels that were associated with student achievement objectives shared across the state assessments in Delaware, Maryland, and Pennsylvania. These topics spanned number and operations, geometry, measurement, probability, data analysis, and algebra (including patterns and functions). Within the upper-elementary and the middle-grades bands, the distribution of these shared student objectives by content topic in the most recent description of each state's mandated assessments for students was remarkably similar, sharing the distribution released for the National Assessment of Educational Progress and Trends in International Mathematics and Science Study assessments at the fourth- and eighth-grade levels. Because the intent was for teachers to complete all assessment and survey instruments in a single day, each teacher-knowledge assessment

Table 2
Demographics and Professional Context of Participating Teachers

Characterizations of teachers	Grades 4 and 5 (*n* = 259)	Grades 6, 7, and 8 (*n* = 184)
Gender (%)		
Female	86.9	78.3
Race/Ethnicity (%)		
White	80.7	75.0
Black/African American	13.9	17.9
Hispanic	2.3	1.6
Asian, Asian American, or Pacific Islander	1.2	3.8
Others (Native American or Multiracial)	1.9	1.6
Certification/Highest degree earned (%)		
Not certified	6.2	8.7
Certified, only holding a bachelor's degree	49.0	49.5
Certified, and subsequent master's degree	20.9	17.9
Certified through a master's degree program	23.9	23.9
Mean number of mathematics/mathematics education courses (*SD*)		
Mathematics courses	2.4 (1.3)	5.2 (3.9)
Mathematics education courses	1.0 (.8)	1.2 (1.1)
Mean years of teaching experience (*SD*)	3.4 (1.6)	3.7 (1.7)
School district location (%)		
Large city	37.8	35.9
Suburb	31.3	30.4
Midsize or small city	21.2	20.1
Small town or rural	9.7	13.6
Special education certification (%)	16.6	20.1
Secondary mathematics certification (%)	---	53.8
Only taught students in Grade 6 (%)	---	26.6
Only taught mathematics during 2008–2009 (%)	6.6	70.7
Taught some students an above-grade mathematics curriculum (%)	32.0	65.8

Table 3
Demographics of Students Taught Mathematics by Participating Teachers

Characteristics of students	Grades 4 and 5 (*n* = 6,413)	Grades 6, 7, and 8 (*n* = 10,890)
Gender (%)		
Female	49.8	49.9
Race/Ethnicity (%)		
White	35.0	37.9
Black/African American	45.5	45.3
Hispanic	15.0	11.2
Asian, Asian American, or Pacific Islander	3.7	5.3
Others (Native American or Multiracial)	.8	.3
Special education students (%)	14.5	11.6
English language learners (%)	5.1	3.4
Students with poverty indicator[a] (%)	56.8	55.1

[a]In one district, only school-level data for free and reduced-price meals were available.

was limited to 120 multiple-choice items (80 CK and 40 PCK items). Sample items are presented in Figure 3.

CK items. The operating principle was that although the CK items had to be related to student content standards in each of the three states, the mathematical understandings measured through those items would not be expectations presumed of students. Rather, the CK items were to measure teachers' deep understanding of the mathematics related to or underlying the content specified in the framework.

For example, eighth-grade mathematics teachers in each of the three states were expected to teach students about characteristics of linear functions, as defined by their state curriculum, addressing slope as well as symbolic form and graphical representation and orientation (e.g., how a linear function's graph and algebraic definition convey whether that linear function is increasing or decreasing over an interval, the relative steepness of its slope, and function values for a given value of x.) No items measuring only this content were included on the assessment of middle-grades teachers' CK, as that understanding was presumed. However, the first sample CK item in Figure 3 displays a more advanced item that is mathematically related to the linear function content in the states' middle-grades student standards and assessments.

The distribution of the 80 CK items over the six mathematical topics reflected the distributions in the shared student objectives of each grade band. To ensure a range of difficulty and complexity in the CK items for teachers, an intended item

CK Item for Middle-grades Teachers (Level 3; Patterns, Functions, and Algebra)

Consider the following functions:

$f(x) = 2x$ $g(x) = x^2$ $h(x) = 2^x$

What can be said of the graphs of the functions, f, g, and h?

*A. For $x > 0$, all three functions are increasing with $h(x)$ increasing at the greatest rate. For $x < 0$, only $f(x)$ has negative values.

B. For $x > 0$, all three functions are increasing with $h(x)$ and $g(x)$ increasing at the same rate. For $x < 0$, only $f(x)$ has negative values.

C. For all real numbers, x, all three functions are always increasing with $h(x)$ increasing at the greatest rate. For $x < 0$, both $h(x)$ and $f(x)$ have negative values.

D. For all real numbers, x, all three functions are always increasing with $h(x)$ and $g(x)$ increasing at the same rate. For $x < 0$, both $h(x)$ and $f(x)$ have negative values.

CK Item for Upper-elementary Teachers (Level 2; Data Analysis)

Two students from the same class moved out of state. Those students' scores in mathematics were equal to the class mean. How does their leaving affect the distribution of class mathematics scores?

A. The mean decreases, but the range of scores stays the same.

B. The mean does not change, but the range of scores decreases.

*C. The mean stays the same, and the range of scores stays the same.

D. It cannot be determined whether the mean changes, but the range of scores stays the same.

PCK Item for Middle-grades Teachers (Domain 1; Number and Operations)

Mr. Croninger's sixth-grade class is working on the word problem below.

> If it takes 3 painters 5 hours to paint the bedrooms and living room in a house, how long would it take 6 painters to paint those same bedrooms and the living room? (Assume all painters are equally skilled.)

What is (are) common misconception(s) held by students that lead to error when solving a problem such as this?

I. Setting up a direct proportion of painters to hours

II. Treating this as an additive increase (3 more painters means 3 more hours)

III. Finding the rate of painters per hour (3/5) and multiplying by 6 painters

A. I only

B. II only

*C. I and II only

D. II and III only

PCK Item for Upper-elementary Teachers (Domain 3; Measurement)

Students in the upper-elementary grades learn to compute the area of parallelograms, rectangles, trapezoids, and triangles. In which order should students consider the area of shapes so that they can derive the formulas and see their connections?

*A. rectangle, parallelogram, triangle, trapezoid

B. triangle, rectangle, parallelogram, trapezoid

C. rectangle, trapezoid, triangle, parallelogram

D. triangle, rectangle, trapezoid, parallelogram

Figure 3. Sample CK and PCK items from the teacher knowledge instrument. Correct answers are marked with an asterisk.

distribution reflecting three of Webb's (2002) levels of "depth of knowledge" was specified. As defined by Webb, *recall items* (Level 1) presume direct knowledge of a fact, definition, term, or a simple procedure as well as skill in performing a simple algorithm or applying a formula. *Skill and concept items* (Level 2) involve more than the recall of a habituated response, as these items may require the solver to make a decision, to recognize the need to organize information or to apply a procedure or definition in an unfamiliar setting, or to solve a multistep problem. *Strategic thinking items* (Level 3) require reasoning, relating ideas, making connections, drawing conclusions, using concepts, or offering explanations of thinking.

We reviewed mathematics education resources and assessment literature and identified released teacher-knowledge items that we then coded by framework objective and depth of knowledge. Items that aligned with framework entries were subsequently modified to fit a multiple-choice format. Then multiple-choice mathematics-content items were written to fill gaps in the item-to-framework alignment in order to yield at least two items for every cell in the intended item-distribution matrices for the upper-elementary and the middle-grades teacher assessments. The research team screened and edited all items; then the emerging item pools were separated into smaller subsets of items, as determined by mathematical topic and grade band. Each subset of items was sent to two mathematics educators and one mathematician (from a pool of seven mathematics educators and five mathematicians) for external vetting.

Reviewed content items were then modified or rewritten, yielding more than 320 items for piloting. These items were clustered into one of eight topic-related subsets for piloting (addressing number and operations, geometry and measurement, data analysis and probability, or patterns, functions, and algebra, as designed for teachers of Grades 4 and 5 or for teachers of Grades 6, 7, and 8). Subsequently, 29 to 34 teachers from a pool of 97 in-service and 17 preservice teachers completed each of these subtests. Following completion of classical test theory procedures (reliability, item difficulty, distracter analysis, point-biserial correlation), two collections of 80 CK items were finalized, one for upper-elementary teachers and one for middle-grades teachers. The distribution of CK items across mathematical content topics by levels is presented in Table 4; refer to Figure 3 for sample CK items.

PCK items. In a review of the literature we located some open-ended, released PCK items, most of which addressed understanding of students' interpretations of mathematics (Domain 4) or student errors and misconceptions (Domain 1). We developed additional multiple-choice PCK items that reflected well-established principles or documented research for practice and that addressed the teaching of mathematical content identified for a given grade band. Following screening and editing, sets of items designed for either upper-elementary or for middle-grades teachers were each sent to three mathematics educators and one school district mathematics supervisor (from a pool of six mathematics educators and two mathematics supervisors) for external vetting. In addition, attendees at a work session

Table 4
Number of CK Items Distributed Across Mathematical Content by Level

	Grades 4 and 5			Grades 6, 7, and 8		
Mathematical content	Level 1	Level 2	Level 3	Level 1	Level 2	Level 3
Number and Operations	10	15	5	7	12	5
Geometry	3	4	2	4	6	2
Measurement	4	5	2	2	3	1
Probability	1	2	1	2	3	1
Data Analysis	2	6	2	3	5	2
Patterns, Functions, and Algebra	5	8	3	7	11	4
Total	25	40	15	25	40	15

held during an NCTM Research Presession reviewed selected PCK items. Accessing reviews from all sources, 42 upper-elementary and 43 middle-grades PCK items were revised prior to pilot testing by 72 practicing teachers. These items were then subjected to classical test theory procedures, yielding two collections of 40 PCK items, one for administration to upper-elementary teachers and one for administration to teachers of middle-grades mathematics. Table 5 presents the distribution of the PCK items across mathematical topics by domains; refer to Figure 3 for sample PCK items.

Table 5
Number of PCK Items Distributed Across Mathematical Content by PCK Domains

	Grades 4 and 5				Grades 6, 7, and 8			
	Domain				Domain			
	1	2	3	4	1	2	3	4
Number and Operations	7	3	1[a]	4	5	2	1	4
Geometry	2	0	1	2	3[a]	0	0	3
Measurement	1	1	1	2	0	0	1	2
Probability	1	0	0	1	2	0	0	1
Data Analysis	2	1	0	3	2	1	0	2
Patterns, Functions, and Algebra	3	0	2	2	4	2	2	3
Total	16	5	5	14	16	5	4	15

[a]One item in each of these cells was deleted from the analysis as a result of IRT scaling.

Item response theory (IRT) scaling. Following test administration, an exploratory factor analysis was performed on the 80 CK items and 40 PCK items of the upper-elementary and the middle-grades instruments to examine the reliability of each instrument and to determine whether subsets of items within each instrument tapped into a single underlying knowledge construct. This analysis verified that each instrument was separately assessing two different dimensions of teacher knowledge, CK and PCK.

IRT analyses indicated that a two-parameter model was appropriate for these data. This process takes into account not only the proportion of correct responses but also the level of item difficulty and the relationship of an item to the construct being measured. The IRT analyses identified two inconsistent PCK items: one item that was administered to the upper-elementary teachers and one item that was administered to the middle-grades teachers. These items were removed prior to determining the upper-elementary and the middle-grades teachers' IRT-scaled CK scores, IRT-scaled PCK scores, and IRT-scaled total teacher knowledge scores (scores across the 119 CK and PCK items). The individual teacher scores are expressed in standard deviations, with mean 0 and standard deviation 1. The empirical reliability value of the 119 items within each of the teacher knowledge measures was .932 (upper elementary) and .941 (middle grades). The empirical reliability values for the 80-item CK and 39-item PCK measures respectively were .925 and .704 (upper elementary) and .930 and .752 (middle grades).

Beliefs and awareness survey. We developed and administered a survey composed of Likert-format items addressing teachers' mathematics teaching and learning beliefs and teachers' claimed awareness of student dispositions. This survey presented the most efficient method for collecting one-point-in-time data measuring the perceptions of a large number of teachers. Instruments with items in the Likert format continue to be one of the most commonly used formats in contemporary survey design and survey research (Babbie, 2010) and are viewed as an acceptable means for testing quantitative hypotheses, particularly when the survey is limited to a single administration.

In a study of teacher's implementation of standards-based reform, Ross, McDougall, Hogaboam-Gray, and LeSage (2003) developed a 20-item instrument measuring teachers' beliefs about mathematics teaching and learning via Likert-format items. Factor analysis utilizing data from administration of this survey to a large sample of elementary teachers in Virginia (Campbell & Malkus, 2010) identified two orthogonal factors within this instrument as well as many items that did not load on any factor. Since Ross et al. (2003) designed their instrument, educational assessment policies have shifted. These changes may have led to decreasing variance in teachers' responses on this Likert-format survey. For example, one item in the Ross et al. survey stated, "I teach students how to explain their mathematical ideas" (p. 349). Because state assessments in the mid-Atlantic region ask elementary students to compose brief constructed responses, there was concern that teachers being sampled for this study would be likely to agree with this statement, as was found with teachers from Virginia. In order to update the items in the instrument, the research team reworded many of the Ross et al. (2003) items to respond to concerns identified by Campbell and Malkus (2010) and defined new items to reflect current dynamics in education.

The resulting beliefs items spanned five categories of mathematics teaching and learning: (a) focus of mathematics classroom instruction, (b) how instruction should be ordered and classroom and materials should be organized, (c) how students learn mathematics best, (d) the role of students in the mathematics classroom, and (e) the role of the teacher in the mathematics classroom. The intent was not to measure the extent to which teachers were aligned with a theory of instructional practices or a theory of learning but rather to detect any underlying factor structure and how different factors emerged across the categories. The final 30 beliefs items included four items worded as published by Ross et al. (2003) and four items modified from their survey.

A literature review addressing students' mathematical dispositions (Kilpatrick et al., 2001) and identities (Martin, 2000) yielded three dimensions within which teachers' awareness of their students varied: (a) awareness of students' self-perceptions of mathematics ability, (b) awareness of students' perspectives regarding the importance of mathematics, and (c) awareness of students' motivations to perform in mathematical contexts. Ten Likert-format items were developed asking teachers to assess the degree to which they had a sense of their students' outlook or gathered explicit information to determine this awareness along each of the three dimensions.

An exploratory factor analysis was completed using 459 teachers' responses on the 40-item beliefs and awareness survey. This analysis was conducted in order to explore the underlying dimensions of teacher's beliefs and awareness and to allow items to migrate, thereby defining factors. The choice to allow items to migrate emerged from the expectation that as the conceptualization of mathematical proficiency becomes more complex (Kilpatrick et al., 2001), teachers' belief systems will also become more complex. Allowing for migration of items, therefore, allowed for the clustering of items in new ways. Examination of the Scree plot yielded three factors. An oblique rotation verified that the correlations between factors were small; a subsequent varimax rotation yielded item loadings. The rotated solution, using the criteria of the absolute value of an item loading .4 or above on only one factor, yielded three interpretable orthogonal factors, two beliefs factors, and one awareness factor.

One factor reflected the belief that teachers should allow students to struggle or grapple with solving problems on their own before teacher intervention (Cronbach's alpha of .662). This belief is aligned with Hiebert and Grouws's (2007) description of a key feature in teaching mathematics focused on students' conceptual understanding and meaning making. They used the word "*struggle* to mean that students expend effort to make sense of the mathematics, to figure something out that is not immediately apparent" (p. 387).

The second factor reflects the belief that teachers should model activities and approaches followed by student practice, emphasizing incremental mastery of procedural skills prior to solving application problems (Cronbach's alpha of .653). This belief is aligned with what Battista (2001) termed the *universal script*—that is, where teachers explicitly demonstrate a procedure or skill for students, and students replicate the demonstrated skill through repetition and practice. It should be noted that the first two factors extracted are not and should not be conceptualized as pedagogical opposites because it is possible for teachers to hold both beliefs strongly and simultaneously.

The third factor reflected the extent to which teachers claimed to know about their students' mathematical dispositions, take explicit actions to learn about their students' mathematical dispositions, highlight multiple approaches to solving a problem during instruction, and include problems that have multiple solutions in their instruction (Cronbach's alpha of .675). Figure 4 displays a sample of items for each of the three factors. Each of these factors met the criteria for reliability of .650 (DeVellis, 2003).

Professional background and instructional context surveys. Two surveys were created to gather information on the teachers' professional background and instructional contexts. On the professional background survey, teachers provided information identifying their certification status and type, route to certification, education level, titles of completed mathematics content and mathematics education courses, and years of teaching experience. Teachers were asked to bring unofficial copies of their transcripts to the data collection site for personal reference when completing the section of the survey addressing course titles. Teachers noted their teaching assignments during 2008–2009 on the instructional context survey.

Analysis and Results

To determine whether there was a relationship between teachers' mathematical content and pedagogical knowledge, teachers' perceptions, and student achievement, we used Hierarchical Linear Modeling (HLM) with a two-level, random intercept model. HLM methods were utilized because of the hierarchical structure of the data associated with students nested within a teacher. Because the teacher-knowledge measures for upper-elementary and middle-grades teachers were distinct, teacher and linked-student data were analyzed by grade-band assignment. The individual upper-elementary and middle-grades student mathematics achievement data, as measured on the state tests, were standardized separately within each state achievement data set; then, within each grade band, the student data were standardized across the achievement data sets from all three states. This was done to make the scores comparable across the states and then across the predictors and control variables in the HLM analysis. This approach yielded distinct upper-elementary and middle-grades, standardized, student achievement data sets.

The intent was to examine the possible relationship between student achievement and teachers' mathematical content and pedagogical understanding, but computed Pearson's correlation coefficients indicated that teachers' CK and PCK scores were

Factor: Teacher Allowance for Student Struggle With Problems
During mathematics class, students should be asked to solve problems and complete activities by relying on their own thinking without teachers modeling an approach.

Students can figure out how to solve many mathematics problems without being told what to do.

Factor: Teacher Modeling for Incremental Mastery
Students learn mathematics best by paying attention when their teacher demonstrates what to do, by asking questions if they do not understand, and then by practicing.

Mathematics skills are mastered incrementally, so instruction should only focus on one skill at a time, ordered by difficulty, and not move on until most students have mastered that skill.

Factor: Teachers' Claimed Awareness of Their Students' Mathematical Dispositions
I learn about my students' perceptions of what "doing mathematics" means through explicitly asking them (e.g., students write about it, one-on-one discussions, group discussions).

For the majority of my students, I have a good sense of their motivations for wanting to succeed in mathematics.

I like to use mathematics problems that can be solved in many different ways.

Figure 4. Sample items from teacher perception (beliefs and awareness) instrument.

correlated (r = .676 for upper-elementary teachers' scores, $p < .001$; r = .741 for middle-grades teachers, $p < .001$). Thus, for each grade band, separate analyses were conducted to investigate the relationship between CK and student achievement and between PCK and student achievement. For each grade band, an additional HLM analysis using combined teachers' scores on both the CK and PCK items within the entire teacher-knowledge measure (TK) was also completed.

There was a statistically significant correlation between student race or ethnicity and the poverty indicator (Pearson's correlation of .415 and .404 for upper-elementary and middle-grades students, respectively). Thus only one of these variables could be entered as a student identifier. A prior study indicated that teachers in schools with higher levels of student poverty performed more poorly on measures of mathematics knowledge for teaching, as compared to teachers from schools with lower levels of poverty. Further, that study found that teachers in schools with a higher proportion of Hispanic students had lower scores on their teacher knowledge assessment (Hill & Lubienski, 2007). Because "students living in poverty" was a characteristic across the school districts sampled in this study and because over 85% of this study's students were not Hispanic, these HLM analyses used the indicator of poverty, rather than race or ethnicity, as a student-level variable.

Because of the hierarchical structure of the data, a portion of the variance in students' achievement scores can be attributed to a student's teacher rather than to individual differences. The interclass correlation coefficient (ICC) measures the proportion of the total variance in students' scores occurring between teachers rather than the variance in student scores within teachers. The ICC measures, resulting from the fully unconditional models, indicated that 30.8% of the variance in the upper-elementary scores and 37.5% of the variance in the middle-grades scores were associated with teacher assignment. Because neither of these percentages was close to 0, the use of a two-level model for each analysis was warranted.

Teachers' CK Analysis

Models. The two-level model partitions the variance in student mathematics achievement into two components: variance between students taught by their respective teachers (Level 1) and variance between the teachers themselves (Level 2). The student-level model for both the upper-elementary and the middle-grades analyses (shown below) included controls for gender (Female), special-education status (SpecEd), a poverty indicator (Poverty), English-language-learner status (ELL), and student's standardized mathematics achievement score on the prior year's (2007–2008) state assessment (PriorStAch). The variable PriorStAch was standardized by grade band with mean 0 and standard deviation 1. All remaining student-level variables were treated as dichotomous indicators, with a value of 1 indicating that the student had the characteristic listed. Because students' achievement scores on the prior year's state assessment were correlated with other predictors and because this relationship may indicate the presence of multicollinearity, the variable PriorStAch was group-mean centered. The remaining student variables were centered on the grand mean, controlling for these student characteristics across both levels. The Level 1 model was:

$$Y_{ij} = \beta_{0j} + \beta_{1j}(\text{Female})_{ij} + \beta_{2j}(\text{SpecEd})_{ij} + \beta_{3j}(\text{Poverty})_{ij} + \beta_{4j}(\text{ELL})_{ij} + \beta_{5j}(\text{PriorStAch})_{ij} + r_{ij}.$$

In this model, the dependent variable Y_{ij} represents the mathematics achievement score of student i with teacher j; β_{0j}(intercept) represents the mean 2008–2009 mathematics achievement of the students taught by teacher j; β_{nj} (n = 1, 2, 3, 4, 5) are the Level 1 coefficients that measure the effects of the five student characteristics respectively on individual student achievement; and r_{ij} is the unique effect of student i on achievement.

Multiple imputation was utilized to estimate the prior achievement scores for 595 upper-elementary students (9.3% of the total number of students in the analytic sample for Grades 4 and 5) and 1,056 middle-grades students (9.7% of the total number of students in the analytic sample for Grades 6, 7, and 8) because these data were missing due to student mobility. Through multiple imputation (Rubin, 1996; Schafer, 1999), five sets of data were generated with those students who originally had missing scores now having imputed prior achievement scores. Then regression results utilizing each of the five data sets separately were weighted and pooled. The estimates produced by this multiple imputation method are considered to be more accurate than those produced by a single imputation procedure. The imputation process used as many relevant predictors as available in the data to calculate best estimates of the prior achievement scores. These included accessible 2007–2008 student scores, students' 2008–2009 scores, grade level, gender, race, school-district identification, an indicator flagging students more than 2 years older than typical for their grade, and student status in terms of special education, English language learner, and poverty.

Upper-elementary teachers. The full teacher-level model for analysis of the upper-elementary data included variables associated with teachers' knowledge and perceptions as well as their professional background and instructional assignment. In this model, the variable CK denotes a standardized IRT-scale score on the 80 mathematical content items developed for upper-elementary teachers. Teachers' perceptions were reflected in three variables, indicating scores on those items of the beliefs and awareness survey that loaded on one of three factors. These include the belief that teachers should allow students to struggle and grapple with solving problems prior to teacher intervention (AllowStrgle) and the belief that teachers should model how to complete mathematical tasks, organizing instruction to support incremental mastery of skills (ModIncreMstry). The third variable (Aware) indicates scores on those survey items identifying the extent to which teachers claimed awareness of their students' mathematical dispositions and the extent to which teachers claimed to highlight multiple approaches to solving a problem during instruction and to include problems that have multiple solutions in their instruction. Because this study's conceptual model recognizes that teachers' knowledge and beliefs may interrelate to influence instructional practice, three Level 2 interaction variables were included (CK × AllowStrgle, CK × ModIncreMstry, and CK × Aware) to model the interactions between knowledge and beliefs.

The professional background variables included number of years of teaching experience (Exp) and a dichotomous indicator of a special-education certification or endorsement (SpEdCert). The value of the teaching-experience variable was defined as the number of years of teaching experience minus 1, in order to establish teachers with 1 year of experience as the reference group. The remaining Level 2 variables in the analysis of upper-elementary data addressed teaching assignment. AbvGL and MthOnly were each dichotomous variables, respectively indicating whether a teacher was teaching an above-grade-level mathematics curriculum to some students, such as teaching the school district's Grade 6 curriculum to Grade 5 students, and whether a teacher specialized in teaching only mathematics each day.

In order to understand the degree to which the variables associated with teacher content knowledge, perceptions, professional background and teaching assignment, and the interactions of teacher knowledge and perception variables explained the variance in student achievement, analyses employing differing HLM models were completed. All teacher variables in the full Level 2 model shown below were grand-mean centered with fixed effects.

$$\begin{aligned}\beta_{0j} = \gamma_{00} &+ \gamma_{01}(\text{CK})_j + \gamma_{02}(\text{AllowStrgle})_j + \gamma_{03}(\text{ModIncreMstry})_j \\ &+ \gamma_{04}(\text{Aware})_j + \gamma_{05}(\text{Exp})_j + \gamma_{06}(\text{SpEdCert})_j + \gamma_{07}(\text{AbvGL})_j \\ &+ \gamma_{08}(\text{MthOnly})_j + \gamma_{09}(\text{CK} \times \text{AllowStrgle})_j \\ &+ \gamma_{010}(\text{CK} \times \text{ModIncreMstry})_j + \gamma_{011}(\text{CK} \times \text{Aware})_j + u_{0j}.\end{aligned}$$

The dependent variable β_{0j} is the average standardized mathematics achievement score of students taught by teacher j; γ_{00} is the average standardized mathematics achievement score of students across teachers; γ_{0n} are the Level 2 coefficients that measure the effects of the 11 independent variables and interaction terms on average student achievement, with these variables estimating teacher content knowledge ($n = 1$), teacher perceptions ($n = 2, 3, 4$), professional background and teaching assignment ($n = 5, 6, 7, 8$), and interactions ($n = 9, 10, 11$).

Middle-grades teachers. The full teacher-level model for analysis of the middle-grades data deleted the variable MthOnly, which was included in the Level 2 upper-elementary model, because teacher assignment within content departments was typical in the middle grades. All other entries in the Level 2 and full model for the analysis of middle-grades teacher and student data remained as in the models for the analysis of upper-elementary data.

Results. Findings from the analyses of the upper-elementary and middle-grades student achievement data with CK as the measure of teachers' mathematical knowledge are addressed first, followed by a discussion of findings with PCK as the measure of teachers' knowledge. Table 6 presents findings from the CK analyses with the statistics for independent variables presented in each row and the grouped columns specifying grade bands. For each grade band, three differing models are noted. Model 1 includes all student-level variables and a single, teacher-knowledge variable; Model 2 adds the teacher-level variables associated with the main effects for teachers' beliefs and awareness. Model 3 presents the results from the final analyses limited to, for the sake of parsimony, all main effects (regardless of whether or not they were statistically significant), including the variables associated with teachers' professional background and instructional assignment and only those interaction terms that had statistically significant effects on achievement.

HLM allows for partitioning of the total variance in students' achievement scores into two components: within-teacher variance, or variance associated with differences between students taught by their respective teachers, and between-teacher variance, or variance associated with differences between students

Table 6
Effects of Teacher Content Knowledge and Perceptions on Student Mathematics Achievement by Grade Band

	Grades 4 and 5: Unstandardized β coefficients (with standard errors)			Grades 6, 7, and 8: Unstandardized β coefficients (with standard errors)		
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
Intercept	-.057 (.035)	-.058 (.035)†	-.059 (.033)†	-.121 (.042)**	-.121 (.041)**	-.123 (.038)**
Student variables						
Female	.013 (.014)	.013 (.014)	.014 (.014)	.018 (.010)†	.018 (.010)†	.018 (.010)†
Special education	-.183 (.036)***	-.183 (.036)***	-.177 (.036)***	-.136 (.028)***	-.135 (.028)***	-.131 (.028)***
Poverty	-.087 (.018)***	-.087 (.018)***	-.087 (.018)***	-.086 (.015)***	-.086 (.015)***	-.085 (.015)***
English language learner	-.049 (.038)	-.048 (.039)	-.047 (.039)	-.003 (.042)	-.003 (.042)	-.003 (.042)
Prior student math achievement	.716 (.014)***	.716 (.014)***	.716 (.014)***	.734 (.014)***	.734 (.014)***	.734 (.014)***
Teacher variables						
Teacher content knowledge	.034 (.033)	.038 (.036)	.071 (.033)*	.222 (.041)***	.210 (.045)***	.166 (.042)***
Allow student struggle		.036 (.044)	-.005 (.043)		.065 (.044)	.068 (.041)
Model for incremental mastery		.003 (.038)	.016 (.035)		.002 (.042)	-.017 (.040)
Aware of student math disposition		.066 (.037)†	.064 (.033)†		.010 (.040)	.007 (.038)
Years of teaching experience			.033 (.021)			.048 (.025)†
Taught above-grade math curriculum			.320 (.079)***			.226 (.082)**
Only taught mathematics in 2008–2009			.139 (.136)			--
Special-education certification			-.191 (.096)*			-.393 (.094)***
Teacher knowledge × Model incremental mastery			--			.102 (.046)*
Teacher knowledge × Aware student disposition			.054 (.022)*			--
	Variance in student achievement by source			Variance in student achievement by source		
Variance estimates						
Student-level variance (s_2)	.316***	.316***	.316***	.264***	.264***	.264***
Teacher-level variance (t_{00})	.304	.303	.274	.320	.321	.273
Reliability estimates (l)	.950	.949	.944	.978	.978	.975

†$p < .10$; *$p < .05$; **$p < .01$; ***$p < .001$

grouped by teachers. Analyses of the variance terms for the CK models indicates that 10% of the between-teacher variance in upper-elementary students' achievement scores can be explained by the teacher characteristics in Model 3, while 26.7% of the between-teacher variance in the middle-grades students' achievement scores can be explained by the teacher characteristics in Model 3.

As indicated in Model 3 in Table 6, there was a statistically significant positive relationship ($\alpha = .05$) between teachers' CK and their students' performance on standardized state mathematics achievement tests for both the upper-elementary and middle-grades students. For each *SD* increase in teachers' CK, the estimated mathematics achievement scores of their students increased 7.1% ($p = .033$) and 16.6% ($p < .001$) of an *SD* at the upper-elementary and middle grades, respectively. This proportion of variance accounted for in student achievement by each *SD* increase in teachers' CK is termed *effect size*. When limiting control variables to only student-level demographics and prior achievement, middle-grades teachers' mathematical content understanding was significantly related to student achievement (effect size of .222, $p < .001$), but this was not the case in the upper-elementary grades (effect size of .034, $p = .298$). For upper-elementary teachers, the relationship between their CK and their students' achievement was not evident until controls for teacher perspectives, professional background, and instructional assignment were entered into the model.

Teachers who taught students a mathematics curriculum identified for a higher grade had students with higher achievement on state assessments designed for the students' actual grade (upper elementary: effect size of .320, $p < .001$; middle grades: effect size of .226, $p = .007$). The upper-elementary and middle-grades students taught by teachers who held certification in special education had statistically significantly lower mathematics achievement scores on the state assessments than did other students (upper elementary: effect size of -.191, $p = .047$; middle grades: effect size of -.393, $p < .001$). This effect was evident even after controlling for the identification of individual students receiving special-education services.

There were grade-band distinctions in the findings addressing student achievement and teachers' perceptions. For upper-elementary teachers, there was a positive association between student achievement and teachers' claimed awareness of their students' mathematical dispositions that did not reach this study's criterion for significance (effect size of .064, $p = .053$). But the analysis indicated a statistically significant effect for the related interaction term CK × Aware (effect size of .054, $p = .014$), revealing a moderating effect by Aware on CK's effect on student achievement. As indicated in Figure 5, students of those upper-elementary teachers with higher levels of CK had achievement that was likely to increase more sharply as teachers' claimed level of awareness increased, although the awareness factor had less effect on the mathematics achievement of students whose teachers had lower CK scores. Consider only upper-elementary students who were taught mathematics by teachers with high CK scores (1 *SD* or more above the mean). If their teachers also had high Aware scores (1 *SD* or more above the mean), then the students were likely to have higher achievement (≥ .24 *SD*) as compared to the students who were taught by the teachers who had low Aware scores (1 *SD* or more below the mean) and high CK scores.

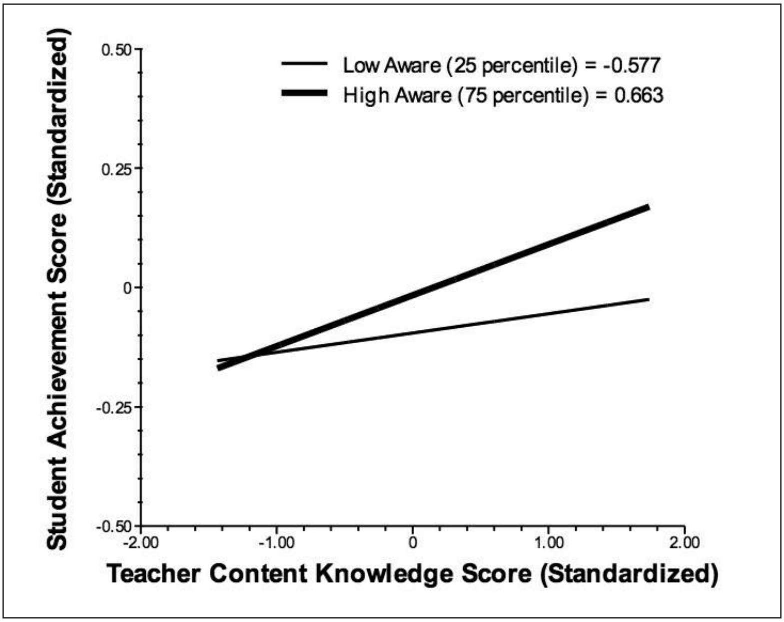

Figure 5. Interaction between upper-elementary teachers' awareness of their students' mathematical dispositions and teachers' mathematical knowledge on student achievement.

Although no statistically significant main effects for variables addressing middle-grades teachers' perceptions were noted in the analyses including teachers' CK, a statistically significant interaction was identified (CK × ModIncreMstry: effect size of .102, $p = .029$). The impact of teachers' CK on student achievement was influenced by teachers' beliefs regarding modeling of solutions to mathematical tasks and organizing instruction to support incremental mastery of skills. As indicated in Figure 6, if middle-grades teachers responded to the beliefs survey items in ways that aligned with the factor represented as ModIncreMstry and they also had knowledge of mathematics content that was lower than the mean CK level, then their students' mathematics achievement was lower than the achievement of students whose teachers had comparable CK and lower scores on the ModIncreMstry factor. As teachers' CK increased, the negative influence of this belief weakened and then reversed. That is, those middle-grades teachers whose CK scores were higher than the mean CK level and who responded to beliefs survey items in ways that aligned with the ModIncreMastry factor had student achievement that was higher than that of students whose teachers had comparable CK and lower ModIncreMstry scores.

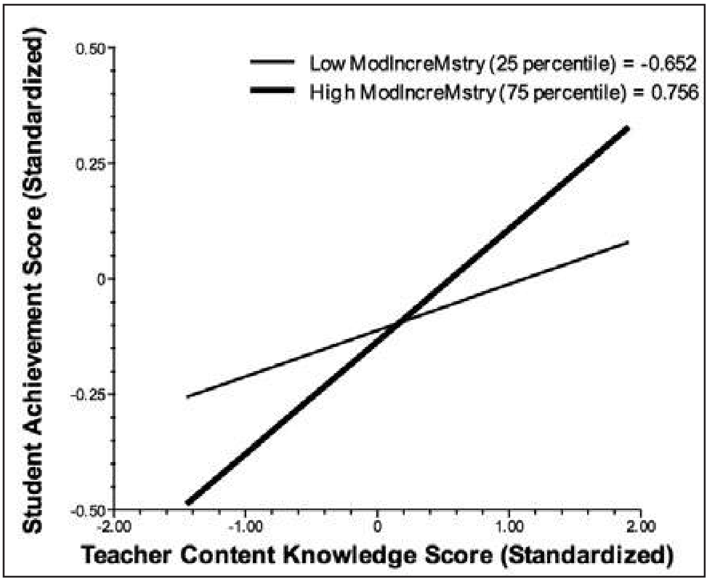

Figure 6. Interaction between middle-grades teachers' belief related to incremental mastery and teachers' mathematical knowledge on student achievement.

Across the upper-elementary and middle grades, the individual effects of poverty and special-education status had statistically significant negative effects on student achievement as measured by state assessments. Students' individual achievement on the prior year's state assessment was predictive, as for each *SD* increase in the 2008 state assessment scores, the estimated student mathematics achievement score on the 2009 state assessment increased over 70% of an *SD*. At the student level, prior 2008 achievement on the state assessment was by far the best predictor of 2009 scores with an effect size exceeding .7.

Teachers' PCK Analysis

In order to examine the relationship between student achievement and teachers' knowledge of mathematics pedagogy, for each grade band, we completed an HLM analysis using teachers' standardized PCK scores. The student-level models for these analyses were as reported previously. The Level 2 and full models for these upper-elementary and middle-grades analyses were as described previously with the exception that, throughout the models, the variable CK was replaced with the variable PCK in both the main effect and all interaction terms. Analysis of the variance terms for the PCK models indicates that 8.8% and 29.2% of the between-teacher variance in upper-elementary students' achievement scores and middle-grades students' achievement scores respectively can be explained by the teacher characteristics noted in the final models of these analyses (Model 3 as reported in Table 7).

Table 7
Effects of Teacher Pedagogical Content Knowledge and Perceptions on Student Mathematics Achievement by Grade Band

	Grades 4 and 5: Unstandardized β coefficients (with standard errors)			Grades 6, 7, and 8: Unstandardized β coefficients (with standard errors)		
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
Intercept	-.057 (.035)	-.058 (.035)	-.059 (.033)†	-.121 (.042)**	-.121 (.042)**	-.123 (.037)**
Student variables						
Female	.013 (.014)	.013 (.014)	.013 (.014)	.018 (.010)†	.018 (.010)†	.018 (.010)†
Special education	-.183 (.036)***	-.183 (.036)***	-.176 (.036)***	-.136 (.028)***	-.135 (.028)***	-.131 (.028)***
Poverty	-.087 (.018)***	-.086 (.018)***	-.087 (.018)***	-.086 (.015)***	-.086 (.015)***	-.085 (.015)***
English language learner	-.048 (.038)	-.048 (.039)	-.047 (.039)	-.004 (.042)	-.004 (.042)	-.003 (.042)
Prior student math achievement	.716 (.014)***	.716 (.014)***	.716 (.014)***	.734 (.014)***	.734 (.014)***	.734 (.014)***
Teacher variables						
Teacher pedagogical content knowledge	.014 (.035)	.017 (.039)	.047 (.038)	.221 (.042)***	.207 (.044)***	.180 (.039)***
Allow student struggle		.041 (.044)	.002 (.043)		.058 (.042)	.055 (.040)
Model for incremental mastery		-.002 (.039)	.008 (.036)		.001 (.042)	-.011 (.040)
Aware of student math disposition		.062 (.038)†	.057 (.035)		.007 (.038)	.011 (.036)
Years of teaching experience			.039 (.021)†			.049 (.024)*
Taught above-grade math curriculum			.310 (.080)***			.289(.078)**
Only taught mathematics in 2008–2009			.140 (.138)			--
Special-education certification			-.175 (.094)†			-.399 (.093)***
Teacher knowledge × Model incremental mastery			--			.138(.052)**
Teacher knowledge × Aware student disposition			--			--
	Variance in student achievement by source			Variance in student achievement by source		
Variance estimates						
Student-level variance (s_2)	.316***	.316***	.316***	.264***	.264***	.264***
Teacher-level variance ($t_{0\beta}$)	.305	.304	.278	.321	.323	.260
Reliability estimates (l)	.950	.950	.945	.978	.978	.973

†$p < .10$; *$p < .05$; **$p < .01$; ***$p < .001$

Examination of the PCK coefficients reported in Table 7 reveals that although teachers' understanding of mathematical pedagogy did not influence their students' mathematics achievement on state assessments in the upper-elementary grades, there was a strong relationship affecting the achievement of middle-grades students. For each *SD* increase in middle-grades teachers' PCK, the estimated mathematics achievement scores of their students increased 22.1% ($p < .001$). These findings persisted even after controls for teacher perspectives, professional background, and instructional assignment were entered into the model. This was the case when the perception variables were modeled either as main effects or as interactions with PCK.

In addition to the expected Level 1 effects of poverty, special-education status, and prior student achievement, the only teacher-level variable statistically significantly related to upper-elementary student achievement when PCK was the measure of teacher knowledge was instructional assignment (AbvGL). On average, student scores on grade-level state achievement tests were higher (effect size of .310, $p < .01$) for teachers who taught some of their students a mathematics curriculum that their school district had identified for students in a more advanced grade.

Regardless of their special-education status, upper-elementary students who were taught mathematics by teachers certified for special education had lower mathematics achievement scores, although this did not reach this study's criterion for significance (effect size of -.175, $p = .064$). In contrast, middle-grades students taught by teachers who held special-education certification had statistically significantly lower mathematics achievement scores on the state assessments than did other students (effect size of -.399, $p < .001$).

The relationship between students' mathematics achievement and the interaction of teachers' knowledge and their belief that mathematics instruction should demonstrate how to complete mathematical tasks in order to support incremental mastery of skills was somewhat stronger when middle-grades teachers' PCK, rather than CK, was the measure of teacher knowledge (PCK × ModIncreMstry: effect size difference of an additional .036). Consider only the middle-grades students who were taught mathematics by teachers with low PCK scores (1 *SD* or more below the mean). If their teachers also had high ModIncreMstry scores (1 *SD* or more above the mean), then the students were likely to have even lower achievement ($\geq .30$ *SD*) as compared to the students who were taught by the teachers who had low ModIncreMstry scores (1 *SD* or more below the mean) and low PCK scores.

When examining the relationship between teachers' PCK and student achievement, for each additional year of teaching experience, up to a maximum of 6 years, there was an increase of approximately 4.9% of an *SD* ($p = .047$) on the estimated mathematics achievement scores of middle-grades students. As noted in Table 7, the effect size associated with the teaching experience of the upper-elementary teachers did not reach this study's .05 criterion for statistical significance when the analysis included teachers' PCK (effect size of .039, $p = .067$).

Teachers' TK Analysis

We also conducted HLM analyses using teachers' standardized scores on the entire teacher knowledge assessment. Other than replacing the variable CK with TK throughout, the Level 2 and full models for these analyses of the upper-elementary and middle-grades student and teacher data mimicked that described for the CK models. Analysis of the variance terms for the TK models indicates that 9.9% and 27.6% of the between-teacher variance in upper-elementary students' achievement scores and middle-grades students' achievement scores, respectively, can be explained by the teacher characteristics included the final model of these analyses (see Table 8 for Model 3 data).

Because 80 of the 119 items on each teacher knowledge assessment were content items, comparison of the findings from the CK (Table 6) and TK (Table 8) analyses reveal remarkably stable estimates with no substantive changes in any of the coefficients except for the teacher knowledge variable in the middle-grades analysis. For middle-grades teachers, the coefficient for TK in Model 3 (effect size of .184, $p < .001$) was larger. Middle-grades teachers' TK was a significant predictor of student achievement when the HLM analysis included only the student-level indicators and TK in the model (effect size of .235, $p < .001$). This was not the case for upper-elementary teachers. There was no finding of a statistically significant relationship between the measure of upper-elementary teachers' TK and their students' performance on state assessments until the complete analytic model (effect size of .071, $p = .041$) was utilized.

The findings of statistically significant interactions in the middle-grades (TK × ModIncreMstry: effect size of .11, $p = .019$) and the upper-elementary (TK × Aware: effect size of .052, $p = .02$) analyses persisted. The positive association between the variable Aware and upper-elementary students' mathematics achievement persisted but still did not reach the criterion level of statistical significance (effect size of .065, $p = .053$).

Discussion

This study's findings addressing the effect of teachers' mathematical content knowledge, pedagogical content knowledge, and perceptions on student achievement in mathematics may inform professional development and teacher education while also serving as an exemplar of how research may be a referent for shaping educational policy. However, because this study relied on a quantitative design, the following aspects were not addressed: how teacher knowledge and perceptions were translated into instructional practice within the teachers' classrooms; how school contexts influenced not only student assignment to teachers but also school culture; how local school administrators and teachers defined instructional priorities; or the duration, quality, local support for, and perceived relevance of, professional development and early-career mentoring experienced by these teachers. Qualitative studies addressing these issues are needed if education research is to contribute to a more nuanced understanding of the role that

Table 8

Effects of Teacher Combined Knowledge and Perceptions on Student Mathematics Achievement by Grade Band

	Grades 4 and 5: Unstandardized β coefficients (with standard errors)			Grades 6, 7, and 8: Unstandardized β coefficients (with standard errors)		
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
Intercept	-.058 (.035)	-.058 (.035)†	-.059 (.033)†	-.121 (.041)**	-.121 (.042)**	-.123 (.038)**
Student variables						
Female	.013 (.014)	.013 (.014)	.014 (.014)	.018 (.010)†	.018 (.010)†	.018 (.010)†
Special education	-.183 (.036)***	-.183 (.036)***	-.177 (.036)***	-.136 (.028)***	-.135 (.028)***	-.131 (.028)***
Poverty	-.087 (.018)***	-.087 (.018)***	-.087 (.018)***	-.086 (.015)***	-.086 (.015)***	-.085 (.015)***
English language learner	- .049 (.038)	-.048 (.039)	-.047 (.039)	-.003 (.042)	-.003 (.042)	-.003 (.042)
Prior student math achievement	.716 (.014)***	.716 (.014)***	.716 (.014)***	.734 (.014)***	.734 (.014)***	.734 (.014)***
Teacher variables						
Teacher combined knowledge	.034 (.033)	.038 (.037)	.071 (.034)*	.235(.042)***	.226 (.046)***	.184 (.042)***
Allow student struggle		.035 (.044)	-.005 (.043)		.057 (.044)	.060 (.041)
Model for incremental mastery		.003 (.038)	.015 (.035)		.006 (.041)	-.013 (.040)
Aware of student math disposition		.066 (.037)†	.065 (.033)†		.013 (.039)	.012 (.037)
Years of teaching experience			.033 (.021)			.051 (.024)*
Taught above-grade math curriculum			.320 (.079)***			.227(.081)**
Only taught mathematics in 2008–2009			.135 (.137)			--
Special-education certification			-.190 (.096)*			-.389 (.092)***
Teacher knowledge × Model incremental mastery			--			.110 (.046)*
Teacher knowledge × Aware student disposition			0.052 (.022)*			--
	Variance in student achievement by source			Variance in student achievement by source		
Variance estimates						
Student-level variance (s_2)	.316***	.316***	.316***	.264***	.264***	.264***
Teacher-level variance ($t_{0\beta}$)	.304	.303	.274	.314	.316	.266
Reliability estimates (l)	.950	.949	.945	.978	.978	.975

†$p < .10$; *$p < .05$; **$p < .01$; ***$p < .001$

knowledgeable teachers assume and the support that they need. Nevertheless, this investigation provides empirical evidence addressing the relationships between teacher knowledge, teacher perceptions, and student achievement that may inform teacher education for prospective and practicing teachers. Although it is important not to overestimate the implications of this work because no paper-and-pencil assessment of teacher knowledge or teacher perceptions can define mathematics instruction in the classroom, the findings of this study may also inform school-district policies targeting mechanisms for improving teacher effectiveness.

Teacher Knowledge

There was a statistically significant relationship between teachers' TK and student achievement in both the upper-elementary and middle grades, but with upper-elementary teachers this was the case only when other teacher- and student-level controls were included in the model. When the impact of upper-elementary teachers' CK and PCK was examined in separate HLM models with control variables, the size of the effect was not particularly large, although CK had a statistically significant effect on student achievement. This may indicate that other teacher, classroom, or school factors not measured or controlled by the models employed in this study are influencing student achievement on standardized mathematics assessments in the upper-elementary grades.

Concerns about upper-elementary teachers' minimal content knowledge for mathematics have led to calls for specialized teachers for mathematics in the upper-elementary grades, teachers who would teach only mathematics all day to students (National Mathematics Advisory Panel, 2008). One advantage, if this recommendation were to become policy, would be in limiting the scale of teacher enhancement efforts to only specialized teachers. In this investigation we found that schools are already identifying specialized mathematics teachers in the upper-elementary grades, without evidence that students of these teachers performed any differently on the state achievement tests than did upper-elementary students whose teachers taught mathematics and other content each day. Simply implementing a policy that focuses responsibility for upper-elementary students' mathematics instruction on fewer teachers will not ensure that those teachers have a deeper knowledge of mathematics content and pedagogy, nor will it ensure that they are the most effective teachers of mathematics available, even if they are willing to be specialized mathematics teachers in the upper-elementary grades.

For middle-grades teachers, when the analytic model only included teacher CK or PCK along with the student-level indicators, for each *SD* increase in either teachers' mathematical or pedagogical knowledge, the estimated mathematics achievement scores of their students increased by 22% of an *SD*. This is a substantial gain. Middle-grades teachers who understood more mathematics and who understood more about what students think about key mathematical ideas, what misconceptions students might have and why they have them, how to interpret students' emerging mathematical explanations and interpretations, and how to respond to those ideas had students who, on average, evidenced higher mathematics achievement. While this conclusion may at first seem obvious, note that this relationship between teachers' knowledge, both CK and PCK, and student achievement is substantially stronger in the middle grades than in the upper-elementary grades. Further, the middle-grades teachers participating in this study did not uniformly demonstrate these types of mathematical and pedagogical understanding.

These findings have implications for practice. If the intent is to improve student mathematics achievement prior to high school in order to build a necessary base for students' future learning, then a key approach is to enhance the knowledge of their teachers. Indeed, the strength of the effect sizes in these findings indicate that efforts to raise student mathematics achievement in the middle grades will be hampered as long as teachers' mathematical content and pedagogical knowledge remains constant. Teacher preparation coursework and professional development offerings must address both mathematical content and pedagogy in ways that advance teachers' subject-matter understanding and their understanding of students' emerging conceptions of mathematics while also fostering effective instructional skills and practices. This may require linked offerings that are marked by coherence, focus, and specificity, referencing expectations for students (Georges, Borman, & Lee, 2010) while supporting teachers' efforts to adapt to local contexts and student needs.

We also found that the students of upper-elementary and middle-grades teachers who held certification in special education had statistically significantly lower performance on their state's achievement tests. This negative relationship—which was particularly strong for middle-grades students—did not emerge merely because special-education teachers teach students coded as qualifying for special-education services; that was controlled through the student-level model. Although there are reports that teachers with special-education licensure frequently hold distinctly different pedagogical stances, as compared to other certified teachers (Boyd & Bargerhuff, 2009), this finding indicates that there are also differences in how special-education teachers' content and pedagogical knowledge relates to student achievement.

This finding confirms that of Neild, Farely-Ripple, and Byrnes (2009), who reported that, even after controlling for student characteristics, middle-grades students taught mathematics by teachers with certification in special education had significantly less growth on a state's mathematics achievement tests than did students taught by either elementary-certified or secondary-mathematics-certified teachers. They also noted that many of the special-education teachers in their middle-school sample were teaching mathematics to students who were not classified for special-education services but who "had a math teacher whose best qualification for teaching mathematics was a special education certificate" (Neild, Farley-Ripple, & Byrnes, 2009, p. 752). This study and the Neild et al. investigation suggest that, at the very least, middle-grades teachers holding certification in special education need more professional development opportunities addressing mathematical content and pedagogy if they are to support their students' learning of mathematics.

Teacher Perceptions

Teachers' beliefs about mathematics teaching and learning and teachers' awareness of their students' mathematical dispositions are thought to influence teaching practices (Wilkins, 2008). This investigation offers evidence that some teacher perceptions actually influence student mathematics achievement, typically as teacher perceptions interact with teacher knowledge.

The main effect for teachers' claimed awareness of students' mathematical dispositions predicted upper-elementary student achievement in mathematics at a level just outside the traditional standard of significance ($p = .053$). At the same time, our analysis revealed a statistically significant interaction effect between upper-elementary teachers' mathematical knowledge and teacher's score on this awareness scale.

Consider those upper-elementary teachers whose mathematical CK or TK scores were at least 1 *SD* higher than the mean score. Why is it that upper-elementary teachers with high mathematical knowledge scores are likely to have even higher student achievement if they are also aware of their students' mathematical dispositions? Does heightened awareness of their students' mathematical dispositions lead these teachers to consider whether their instructional practices are actually promoting understanding for all of their students rather than presuming the effectiveness of their methods? Although further research is needed to clarify the implication of this statistically significant interaction, this finding does highlight that, at least in the upper-elementary grades, student achievement in mathematics is related to teachers' claimed awareness of their students' mathematical dispositions.

The Aware factor did not have this influence at the middle-grades level. However, there were statistically significant interactions between middle-grades teachers' knowledge scores and teachers' belief in the importance of modeling for incremental mastery and their students' mathematics achievement. Middle-grades teachers' mathematical knowledge, both of content and pedagogy, was directly and positively predictive of their students' level of mathematics achievement. But if middle-grades teachers' beliefs about mathematics teaching and learning were also strongly aligned with what Battista (2001) termed the universal script for mathematics instruction, with intended mastery of procedural skills occurring prior to consideration of application problems, then the relationship between teacher knowledge and student achievement was magnified. The mathematics achievement of students whose teachers strongly held this perspective was markedly depressed if their teachers had limited mathematical content or pedagogical content knowledge; at the same time, the achievement of students whose teachers strongly held this perspective was markedly increased if their teachers had strong mathematical knowledge. If students' mathematics achievement is the intended outcome, this investigation finds no evidence to support the assumption that emphasizing mathematical procedure and limiting instructional context to a sequential routine—demonstrate or model, guided practice, and independent practice—will compensate for a middle-grades teacher's weak understanding of mathematics content or pedagogy. Although this finding does not make it any easier to address the challenge of improving teacher quality in order to enhance student achievement, it does point out that the expedient solution of focusing on procedures or even scripting procedurally focused lessons for weaker teachers will be ineffectual.

Final Thoughts

Mathematics teacher educators assume a core responsibility for enhancing the content and pedagogical knowledge of prospective and practicing teachers as well as influencing their beliefs regarding mathematics teaching and learning and their awareness of their students' mathematical dispositions. An implication of this study is that the outcomes of this enterprise affect not only the knowledge and perceptions of teachers but also, ultimately, the mathematics achievement of the students taught by these teachers.

We recognize that our framework for teachers' CK and the data defining student achievement are based on state standards that preceded the Common Core State Standards for Mathematics (CCSSM) (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Yet, we contend that our findings are still relevant, because a comparison of state standards and CCSSM suggest that, for many states, there will be considerable increase in the cognitive demand of mathematical content and of mathematical practices expected across Grades 4–8. This is because of the shifts in grade-specific positioning of mathematics content standards and the depth of mathematical content and practices specified in the CCSSM, particularly in the areas of demonstrating understanding and solving nonroutine problems (Porter, McMaken, Hwang, & Yang, 2011). If the goals of the CCSSM are to be realized, teachers of mathematics will need to deepen their understanding of the mathematics they are expected to teach (Sztajn, Marrongelle, & Smith, 2011). Yet CCSSM implementation goes far beyond curriculum restructuring that specifies the mathematics content students are expected to know; CCSSM implementation also resets expectations for the mathematical processes and proficiencies to be enacted by students in classrooms. In particular, students meeting the Standards for Mathematical Practice are expected to persevere, make sense of problems, construct viable arguments, and critique the reasoning of others. This means not only that mathematics teachers' instructional practices will need to be enhanced and supported but also that teacher beliefs related to mathematics teaching and learning, such as allowing students to struggle and not limiting instruction to an incremental demonstration of mathematical procedures, will need to be examined and discussed in terms of alignment with expectations for students' mathematical practices. This study illustrates the importance and power of teachers' beliefs, particularly as specific beliefs interact with teachers' mathematical knowledge. As such, it provides critical evidence that initiatives focused on CCSSM implementation must balance attention to teachers' mathematical knowledge with attention to teachers' beliefs about mathematics teaching and learning. The increased expectations associated with the CCSSM are likely to place increased demands on teachers and students in the upper-elementary and middle grades, challenging teachers' beliefs about mathematics teaching and learning and heightening the expectations for teachers' mathematical content and pedagogical knowledge as well as their awareness of students' mathematical dispositions.

References

Aguirre, J., & Speer, N. M. (1999). Examining the relationship between beliefs and goals in teacher practice. *Journal of Mathematical Behavior, 18*(3), 327–356. doi:10.1016/S0732-3123(99)00034-6

Babbie, E. (2010). *The practice of social research* (12th ed.). Belmont, CA: Wadsworth.

Ball, D. L. (1991). Research on teaching mathematics: Making subject-matter knowledge part of the equation. In J. Brophy (Ed.), *Advances in research on teaching: Teachers' knowledge of subject matter as it relates to their teaching practice* (Vol. 2, pp. 1–48). Greenwich, CT: JAI Press.

Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 433–456). New York, NY: Macmillan.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal for Teacher Education, 59*(5), 389–407. doi:10.1177/0022487108324554

Battista, M. T. (2001). How do children learn mathematics? Research and reform in mathematics education. In T. Loveless (Ed.), T*he great curriculum debate: How should we teach reading and math?* (pp. 42–84). Washington, DC: Brookings Press.

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., . . . Tsai, Y. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal, 47*(1), 133–180. doi:10.3102/0002831209345157

Beswick, K. (2007). Teachers' beliefs that matter in secondary mathematics classrooms. *Educational Studies in Mathematics, 65*(1), 95–120. doi:10.1007/s10649-006-9035-3

Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 171–200). Westport, CT: Ablex.

Boyd, B., & Bargerhuff, M. E. (2009). Mathematics education and special education: Searching for common ground and the implications for teacher education. *Mathematics Teacher Education and Development, 11*, 54–67. Retrieved from http://www.merga.net.au/documents/MTED_11_Boyd_Bargerhuff.pdf

Boyd, D., Lankford, H., Loeb, S., & Wyckoff, J. (2005). The draw of home: How teachers' preferences for proximity disadvantage urban schools. *Journal of Policy Analysis and Management, 24*(1), 113–132. doi:10.1002/pam.20072

Bray, W. S. (2011). A collective case study of the influence of teachers' beliefs and knowledge on error-handling practices during class discussion of mathematics. J*ournal for Research in Mathematics Education, 42*(1), 2–38.

Bush, W. S., Ronau, R., Karp, K., Thompson, C., Moody, V., Brown, E. T., & McGatha, M. (2005). *An analysis of recommendations regarding the mathematics knowledge expected of elementary and middle school teachers.* Unpublished manuscript, Center for Research in Mathematics and Science Teacher Development, University of Louisville, Louisville, KY.

Campbell, P. F., & Malkus, N. N. (2010). The impact of elementary mathematics specialists. *Journal of Mathematics and Science: Collaborative Explorations, 12*, 1–28. Retrieved from http://www.math.vcu.edu/g1/journal/Journal_12/1_Campbell_Malkus

Conference Board of the Mathematical Sciences. (2001). *The mathematical education of teachers: Issues in mathematics education* (Vol. 11). Providence, RI: American Mathematical Society.

Cooney, T. J., Shealy, B. E., & Arvold, B. (1998). Conceptualizing belief structures of preservice mathematics teachers. *Journal for Research in Mathematics Education, 29*(3), 306–333.

Darling-Hammond, L., & Sykes, G. (2003, September 17). Wanted: A national teacher supply policy for education: The right way to meet the "highly qualified teacher" challenge. *Education Policy Analysis Archives, 11*(33). Retrieved from http://epaa.asu.edu/ojs/article/viewFile/261/387

DeVellis, R. (2003). *Scale development: Theory and applications* (2nd ed.). Thousand Oaks, CA: Sage Publications.

Educational Testing Service. (2008). *Study guide for the middle school mathematics test*. Princeton, NJ: Author. Retrieved from: http://www.ets.org/praxis/prepare/materials/0069

Gellert, U. (2000). Mathematics instruction in safe space: Prospective elementary teachers' views of mathematics education. *Journal of Mathematics Teacher Education, 3*(3), 251–270. doi:10.1023/A:1009965408053

Georges, A., Borman, K. M., & Lee, R. S. (2010, June 10). Mathematics reform and teacher quality in elementary grades: Assessments, teacher licensure, and certification. *Educational Policy Analysis Archives, 18*(13). Retrieved from http://epaa.asu.edu/ojs/article/view/757

Goddard, R. D., Hoy, W. K., & Hoy, A. W. (2000). Collective teacher efficacy: Its meaning, measure, and impact on student achievement. *American Educational Research Journal, 37*(2), 479–507. doi:10.3102/00028312037002479

Goldhaber, D. D., & Brewer, D. J. (2000). Does teacher certification matter? High school certification status and student achievement. *Educational Evaluation and Policy Analysis, 22*(2), 129–146. doi:10.3102/01623737022002129

Hiebert, J. S., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 371–404). Charlotte, NC: Information Age.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction, 26*(4), 430–511. doi:10.1080/07370000802177235

Hill, H. C., & Lubienski, S. T. (2007). Teachers' mathematics knowledge for teaching and school context: A study of California teachers. *Educational Policy, 21*(5), 747–768. doi:10.1177/0895904807307061

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(20), 371–406. doi:10.3102/00028312042002371

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *Elementary School Journal, 105*(1), 11–30. doi:10.1086/428763

Ingersoll, R. M. (2001). Teacher turnover and teacher shortages: An organizational analysis. *American Educational Research Journal, 38*(3), 499–534. doi:10.3102/00028312038003499

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics.* Washington DC: National Academies Press.

Love, A., & Kruger, A. C. (2005). Teacher beliefs and student achievement in urban schools serving African American students. *Journal of Educational Research, 99*(2), 87–98. doi:10.3200/JOER.99.2.87-98

Martin, D. B. (2000). *Mathematics success and failure among African-American youth: The roles of sociohistorical context, community forces, school influence, and individual agency.* New York, NY: Lawrence Erlbaum Associates.

Monk, D. H., & King, J. A. (1994). Multilevel teacher resource effects on pupil performance in secondary mathematics and science: The case of teacher subject-matter preparation. In R. G. Ehrenberg (Ed.), *Choices and consequences: Contemporary policy issues in education* (pp. 29–58). Ithaca, NY: ILR Press.

Murphy, P. K., Delli, L. A. M., & Edwards, M. N. (2004). The good teacher and good teaching: Comparing beliefs of second-grade students, preservice teachers, and inservice teachers. *Journal of Experimental Education, 72*(2), 69–92. doi:10.3200/JEXE.72.2.69-92

National Council for Accreditation of Teacher Education/National Council of Teachers of Mathematics. (2003). *NCATE/NCTM program standards: Programs for initial preparation of mathematics teachers: Standards for middle level mathematics teachers*. Reston, VA: Author. Retrieved from http://www.nctm.org/standards/content.aspx?id=2978

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics. Reston*, VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics.* Washington, DC: Author. Retrieved from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel* (ED 00424P). Washington, DC: U.S. Department of Education.

Neild, R. C., Farley-Ripple, E. N., & Byrnes, V. (2009). The effect of teacher certification on middle grades achievement in an urban district. *Educational Policy, 23*(5), 732–760. doi:10.1177/0895904808320675

No Child Left Behind Act of 2001, Pub. L. No. 107-110, § 115, Stat. 1425 (2002). Retrieved from http://www2.ed.gov/policy/elsec/leg/esea02/107-110.pdf

Nye, B., Konstantopoulos, S., & Hedges, L. V. (2004). How large are teacher effects? *Educational Evaluation and Policy Analysis, 26*(3), 237–257. doi:10.3102/01623737026003237

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*(3), 307–332. doi:10.3102/00346543062003307

Peterson, P. L., Fennema, E., Carpenter, T. P., & Loef, M. (1989). Teachers' pedagogical content beliefs in mathematics. *Cognition and Instruction, 6*(1), 1–40. doi:10.1207/s1532690xci0601_1

Philipp, R. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 257–315). Charlotte, NC: Information Age.

Philipp, R. A., Ambrose, R., Lamb, L. L. C., Sowder, J. T., Schappell, B. P., Sowder, L., . . . Chauvot, J. (2007). Effects of early field experiences on the mathematical content knowledge and beliefs of prospective elementary school teachers: An experimental study. *Journal for Research in Mathematics Education, 38*(5), 438–476.

Porter, A., McMaken, J., Hwang, J., & Yang, R. (2011). Common core standards: The new U.S. intended curriculum. *Educational Researcher, 40*(3), 103–116. doi:10.3102/0013189X11405038

Rice, J. K. (2003). The incidence and impact of teacher professional development: Implications for education productivity. In M. L. Plecki & D. Monk (Eds.), *School finance and teacher quality: Exploring the connections* (pp. 111–136). Larchmont, NY: Eye on Education.

Rivkin, S. G., Hanushek, E. A., & Kain, J. F. (2005). Teachers, schools, and academic achievement. *Econometrica, 73*(2), 417–458. doi: 10.1111/j.1468-0262.2005.00584.x

Ross, J. A., McDougall, D., Hogaboam-Gray, A., & LeSage, A. (2003). A survey measuring elementary teachers' implementation of standards-based mathematics teaching. *Journal for Research in Mathematics Education, 34*(4), 344–363.

Rubin, D. B. (1996). Multiple imputation after 18+ years. *Journal of the American Statistical Association, 91*(434), 473–489. doi:10.1080/01621459.1996.10476908

Schafer, J. L. (1999). Multiple imputation: A primer. *Statistical Methods in Medical Research, 8*(1), 3–15. doi:10.1177/096228029900800102

Scott, T. P., Milem, J. L., Stuessy, C. L., Blount, K. P., & Bentz, A. (2006). Math and Science Scholars (MASS) Program: A model program for the recruitment and retention of preservice mathematics and science teachers. *Journal of Science Teacher Education, 17*(4), 389–411. doi:10.1007/s10972-006-9026-3

Sherin, M., Jacobs, V., & Philipp, R. (2011). *Mathematics teacher noticing: Seeing through teachers' eyes*. New York, NY: Routledge.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14. doi:10.3102/0013189X015002004

Smith, P. S., & Esch, R. K. (2012, April). *Identifying and measuring factors related to student learning: The promise and pitfalls of teacher instructional logs*. Paper presented at the annual meeting of the American Educational Research Association, Vancouver, British Columbia, Canada. Retrieved from http://www.horizon-research.com/atlast/wp-content/uploads/Promise-and-Pitfalls-of-Logs.pdf

Stanford Center for Assessment, Learning, and Equity. (2012). *Teacher performance assessment: Secondary mathematics assessment handbook*. Stanford, CA: Board of Trustees of the Leland Stanford Junior University.

Stipek, D., Givvin, K., Salmon, J., & MacGyvers, V. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17*(2), 213–226. doi:10.1016/S0742-051X(00)00052-4

Sztajn, P. (2003). Adapting reform ideas in different mathematics classrooms: Beliefs beyond mathematics. *Journal of Mathematics Teacher Education, 6*(1), 53–75. doi:10.1023/A:1022171531285

Sztajn, P., Marrongelle, K., & Smith. P. (2011, November). *Supporting implementation of the Common Core State Standards for Mathematics: Recommendations for professional development*. Raleigh, NC: North Carolina State University, College of Education. Retrieved from http://www.nctm.org/uploadedFiles/Math_Standards/Summary_PD_CCSSMath.pdf

Szydlik, J. E., Szydlik, S. D., & Benson, S. R. (2003). Exploring changes in pre-service elementary mathematics teachers' mathematical beliefs. *Journal of Mathematics Teacher Education, 6*(3), 253–279. doi:10.1023/A:1025155328511

Wayne, A. J., & Youngs, P. (2003). Teacher characteristics and student achievement gains: A review. *Review of Educational Research, 73*(1), 89–122. doi:10.3102/00346543073001089

Webb, N. L. (2002, March). *Depth of knowledge levels for four content areas.* Unpublished manuscript. Wisconsin Center for Education Research, University of Wisconsin-Madison, Madison, WI. Retrieved from http://facstaff.wcer.wisc.edu/normw/All%20content%20areas%20%20DOK%20levels%2032802.doc

Wilkins, J. L. M. (2008). The relationship among elementary teachers' content knowledge, attitudes, beliefs, and practices. *Journal of Mathematics Teacher Education, 11*(2), 139–164. doi:10.1007/s10857-007-9068-2

Wilson, S. M., Floden, R. E., & Ferrini-Mundy, J. (2002). Teacher preparation research: An insider's view from the outside. *Journal of Teacher Education, 53*(3), 190–204. doi:10.1177/0022487102053003002

Authors

Patricia F. Campbell, Department of Teaching and Learning, Policy and Leadership, University of Maryland, 2226 Benjamin Building, College Park, MD 20742; patc@umd.edu

Masako Nishio, Department of Teaching and Learning, Policy and Leadership, University of Maryland, 2226 Benjamin Building, College Park, MD 20742; hnievemn@gmail.com

Toni M. Smith, American Institutes for Research, 1000 Thomas Jefferson Street NW, Washington, DC 20007; tsmith@air.org

Lawrence M. Clark, Department of Teaching and Learning, Policy and Leadership, University of Maryland, 2226 Benjamin Building, College Park, MD 20742; lmclark@umd.edu

Darcy L. Conant, Mathematics Department, Notre Dame of Maryland University, 4701 North Charles Street, Baltimore, MD 21210; dconant@ndm.edu

Amber H. Rust, Mathematics Department, Anne Arundel Community College, 101 College Parkway, Arnold, MD 21012-1895; arust@umd.edu

Jill Neumayer DePiper, Education Development Center, Learning and Teaching Division, 43 Foundry Avenue, Waltham, MA 02453; jdepiper@edc.org

Toya Jones Frank, Division of Elementary, Literacy, and Secondary Education, George Mason University, Thompson Hall 1800B, 4400 University Drive, Fairfax, VA 22030; toya.jones@gmail.com

Matthew J. Griffin, Department of Teaching and Learning, Policy and Leadership, University of Maryland, 2226 Benjamin Building, College Park, MD 20742; griff23@umd.edu

Youyoung Choi, Hanyang Cyber University, Wangsimni-ro 222 (Haengdang Dong 17), Seongdong Gu, Seoul 133-791, Korea; yountoto@gmail.com

Submitted March 15, 2013

Accepted September 16, 2013