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Teacher support for collective argumentation: A framework for examining how teachers support students' engagement in mathematical activities

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Abstract We propose a framework for examining how teachers may support collective argumentation in secondary mathematics classrooms, including teachers' direct contributions to arguments, the kinds of questions teachers ask, and teachers' other supportive actions. We illustrate our framework with examples from episodes of collective argumentation occurring across 2 days in a teacher's classroom. Following from these examples, we discuss how the framework can be used to examine mathematical aspects of conversations in mathematics classrooms. We propose that the framework is useful for investigating and possibly enhancing how teachers support students' reasoning and argumentation as fundamentally mathematical activities.

Keywords Argumentation · Reasoning · Questioning · Teaching · Discussions

1 Introduction

Many authors have asserted or implied that participating in discussions is helpful for student learning of mathematics (Hufferd-Ackles, Fuson, & Sherin, 2004; Krummheuer, 2000; Stein, Engle, Smith, & Hughes, 2008). Multiple researchers have explored how to facilitate productive mathematical discussions (e.g., Baxter & Williams, 2009; Hufferd-Ackles et al., 2004; Staples, 2007). Participating in discussions in a distinctively mathematical way can be framed as collective argumentation, where collective argumentation involves multiple people arriving at a conclusion, often by consensus. In particular, we examine collective argumentation in classrooms in which the teacher and students work together to establish mathematical claims. In this paper, we introduce the *teacher support for collective argumentation* framework for

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examining how teachers may support collective argumentation in secondary mathematics classrooms. We begin by acknowledging the difficulty of facilitating mathematical discussions, describing recent work aimed to address some of those difficulties, and examining how focusing on collective argumentation can provide even more information about the mathematical importance of such discussions. We then describe our framework for teacher support of collective argumentation[1] and illustrate it with examples from our work with student teachers. Finally, we propose that using the framework allows investigation of the teacher's role in supporting students' reasoning and argumentation as fundamental activities of mathematics.

2 Background

2.1 Facilitating mathematical discussions

Research and accounts of personal experiences suggest that facilitating mathematical discussions is difficult to do well (Hufferd-Ackles et al., 2004; Stein et al., 2008). Lobato, Clarke, and Ellis (2005) suggested that teachers have been hesitant to say too much during these discussions, fearing that could be interpreted as "telling" (p. 101), an action that they saw as having potentially negative consequences (see also Chazan & Ball, 1999). Teachers' hesitation to tell—meaning giving broad hints or actual solutions—often leads to discussions in which the apparent goal is to see multiple solution methods rather than to accomplish a larger mathematical goal. Other authors have suggested that teachers might experience the dilemma of balancing needs of the class against faithfulness to the mathematics (e.g., Brodie, 2010).

As suggested by recent policy documents and recommendations for teachers (e.g., Advisory Committee on Mathematics Education, 2011; Martin, 2007; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), to facilitate productive mathematical discussions, teachers must engage in behavior that helps students build from their own understandings toward appropriate understandings of mathematical ideas. According to Stein et al. (2008), this process begins with choosing appropriate tasks, ones with high cognitive demand. By *high cognitive demand*, Stein et al. mean tasks that engage students in complex thinking processes and have multiple solution paths or entry points. Much has been written about engaging students in high cognitive demand tasks, including having discussions as a whole class after small groups of students work on the tasks, but often the parts of the discussion that are emphasized are the elicitation of multiple solutions or multiple solution paths rather than the connections between mathematical ideas or the mathematical goal of the lesson.

Researchers agree that the teacher plays a pivotal role in orchestrating mathematical discussions, even, or especially, when he or she is not acting as an arbiter of mathematical truth. Staples (2007), in her description of an experienced teacher establishing inquiry practices as normative in her classroom, foregrounded one of the most important aspects of the teacher's role in a collaborative classroom: "The teacher…has considerable influence over whether or not a student's idea is elicited and done so that it can be taken up by the collective" (p. 174). Boaler and Brodie (2004) argued that the questions a teacher asks influence the nature of a

[1] Our framework captures the teacher's actions in support of mathematical arguments in classrooms. It does not distinguish actions that might be mathematically productive from those that might not be. Nor does it distinguish actions that encourage more productive argumentation from others that might limit students' participation. However, it does point out the potential actions and contributions of the teacher and allows users of the framework to draw conclusions about the mathematical and pedagogical potential of such actions.

classroom discussion. Hufferd-Ackles et al. (2004) suggested that the teacher should create a "math-talk learning community" (p. 81) in a classroom as one way to ameliorate the difficulties of facilitating productive discourse, and Stein et al. (2008) described five practices that are useful when facilitating discourse, particularly around cognitively demanding tasks.

Hufferd-Ackles et al. (2004) included four components in their framework for a math-talk learning community: "(a) Questioning, (b) Explaining math thinking, (c) Source of mathematical ideas, and (d) Responsibility for learning" (p. 87). For each of these dimensions, they described four possible levels of the community. Hufferd-Ackles et al.'s framework provided a description of a community in which important mathematical ideas could be discussed. However, their framework does not foreground the importance of appropriate mathematical reasoning or how the community knows when a mathematical idea has been appropriately established. Their category of "explaining math thinking" comes closest in level 3, where students are explaining and defending their thinking, but the actual ideas and reasoning being used at level 3 are not apparent within the categories of their framework.

Stein et al. (2008) began to address the problem of discussions that do not highlight the important mathematical ideas by breaking down the work of the teacher into more manageable parts, which they call the "five practices model" (p. 314). Their model distinctly addressed the problem of concluding a discussion with recognition that there are multiple ways to solve a task, replacing that with a carefully sequenced set of student solutions that culminate with a careful summary that connects the solutions with each other and with the important mathematical ideas that were the intended focus of the task. Stein et al.'s model offloads some of the decision making to the planning stage of teaching, which is very helpful, particularly for beginning teachers. Their practices provide useful guidance for teachers to engage their students in productive discussions. However, we find one element of mathematical thinking not explicitly present in the practices to be important when thinking about moving toward conversations that are mathematically productive. That element is understanding and recognizing a mathematically appropriate argument.

Understanding, recognizing, and constructing mathematical arguments are important parts of the disciplinary practices of mathematics. The National Council of Teachers of Mathematics (NCTM, 2000) emphasized reasoning and proving as well as communicating in their *Principles and Standards for School Mathematics*; their more recent book series describing the necessary components of the high school curriculum is titled *Focus in High School Mathematics*: *Reasoning and Sense Making* (NCTM, 2009), indicating a continued focus on reasoning, which is an essential part of argumentation. The *Common Core State Standards for Mathematics* (*CCSSM*, National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) include among the standards for mathematical practice, "Construct viable arguments and critique the reasoning of others" (p. 7). In addition, the authors of the *CCSSM* assert that being able to justify or derive mathematical statements, an essential part of mathematical argumentation, is an indication of mathematical understanding. One of the goals of these policy documents is to ensure that students engage in practices that are foundational to the discipline of mathematics. Reasoning and proof are unequivocally accepted to be foundational to the discipline of mathematics, but engaging students in formal deductive proof at an early age may not be developmentally appropriate. Argumentation, as a precursor to proof, is fundamental to the establishment of mathematical knowledge.

When analyzing a discussion, teachers and other mathematics educators need to think about the disciplinary practices of mathematics. How did that discussion allow students to participate in the disciplinary practices of mathematics, particularly with regard to the establishment of mathematical claims or the reasoning practices that are important in mathematics? How can teachers foreshadow and eventually lead to thinking about deductive reasoning and proof? We believe focusing on collective argumentation allows mathematics educators to think about what makes a discussion distinctively mathematical. In particular, by separating the different components of an argument, our analysis of collective argumentation allows an examination of the kinds of reasoning used in establishing claims (by examining the warrants in arguments), how the students contribute to that reasoning, and what the teacher is doing to support students in both making and establishing mathematical claims.

2.2 Focusing on collective argumentation

Recent work in mathematics education has highlighted collective argumentation as an important part of classroom discourse. Building on Toulmin's (1958/2003) model of argumentation in multiple fields, mathematics educators, following Krummheuer (1995), have examined collective argumentation in classroom settings. This avenue of research is an extension of Toulmin's work, as he examined argumentation in the traditional sense of one person convincing an audience of the validity of a claim. Some work in mathematics education examines individual construction of arguments (e.g., Hollebrands, Conner, & Smith, 2010; Inglis, Mejia-Ramos, & Simpson, 2007), but we build on studies addressing collective argumentation. Current work in collective argumentation involves examining student learning through that lens (Krummheuer, 2007) as well as examining how ideas become taken "as-if-shared" (Rasmussen & Stephan, 2008, p. 196).

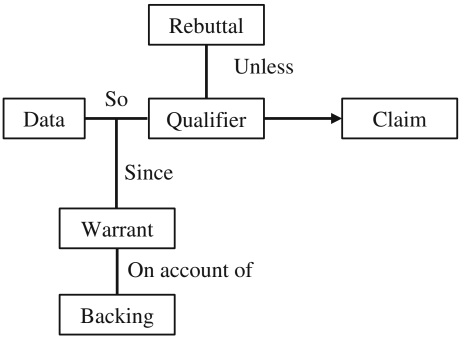
An argument, as described by Toulmin (1958/2003) and currently used in the field, involves some combination of *claims* (statements whose validity is being established), *data* (support provided for the claims), *warrants* (statements that connect data with claims), *rebuttals* (statements describing circumstances under which the warrants would not be valid), *qualifiers* (statements describing the certainty with which a claim is made), and *backings* (usually unstated, dealing with the field in which the argument occurs). Toulmin conceptualized an argument as occurring with a specific structure (see Fig. 1) in which these parts of arguments relate to one another in specific ways. In practice, arguments are often more complicated, in that, for example, statements offered as data may also need defense, thus functioning as both data in one argument and claim in a subargument.[2]

In our research, we respond to the call by Yackel (2002) to examine the teacher's role in collective argumentation by focusing on both the parts of arguments he or she provides and the teaching moves that prompt or respond to parts of arguments provided by students. Our research examines how student teachers, as teachers who are beginning their professional experience, support their students as they engage in collective argumentation.

We define *collective argumentation* very broadly to include any instance where students and teachers make a mathematical claim and provide evidence to support it. An example from our data can be seen in Fig. 2 where a teacher and her students were discussing the characteristics of a regular polygon during a whole-class discussion. They used squares as a prototypical example of regular polygons and made the claim that "the angles are never going to change in a regular polygon, even if the sides do." In our discussion of collective argumentation, we include how teachers support collective argumentation in whole-class discussions as well as how they facilitate small-group discussions. Our definition of collective argumentation builds on a distinction introduced by Toulmin (1958/2003) and emphasized by

[2] Toulmin (1958/2003) called these *preliminary arguments* or *lemmas* (p. 90). We use *subargument* to indicate that these may not come temporally before the other argument and to avoid *lemma*, which has a specific mathematical meaning.

Fig. 1 Diagram of a generic argument (adapted from Toulmin, 1958/2003)



Krummheuer (1995) between "analytic and substantial arguments" (Toulmin, 1958/2003, p. 125). An analytic argument is perhaps best described as one corresponding with proof in mathematics; it is a "logically correct deduction…[that] contains in its conclusion nothing that is not already a potential part of the premises" (Krummheuer, 1995, p. 235), while substantial arguments "expand the meaning of such propositions insofar as they soundly relate a specific case to them by actualization, modification, and/or application" (Krummheuer, 1995, pp. 235–236). We follow Toulmin and Krummheuer in our rejection

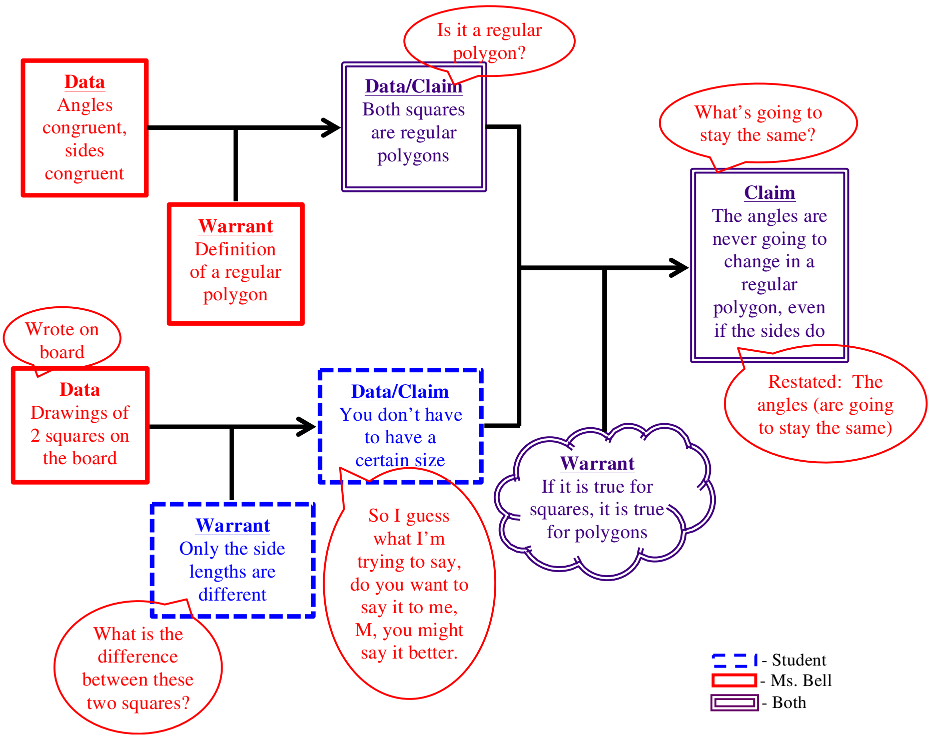


Fig. 2 Example of argument in Ms. Bell's class

of analytic arguments (proofs in mathematics) as the only valid arguments and echo Krummheuer's contention:

> As Toulmin strongly emphasized, a substantial argumentation should not be subordinated or related to an analytic one in the sense that the latter is the ideal type of arguing and that one can always identify in substantial arguments the logical gulf in comparison to an analytic one. Substantial argumentation has a right by itself. By substantial argumentation a statement or decision is gradually supported. This support is not conducted by a formal, logically necessary conclusion, nor by an arbitrary edict such as declared self-evidence, but is motivated by the accomplishment of a convincing presentation of backgrounds, relations, explanations, justification, qualifiers, and so on. (p. 236)

To examine teachers' support for collective argumentation, we modified Toulmin's (1958/2003) model to include teachers' actions that were not directly components of the arguments and to signify who contributed the various components of arguments (as described in Conner, 2008). We use color and line style to indicate whether the teacher, students, or the teacher and students interactively contributed each component of an argument, as seen in Fig. 2 (the teacher contributed the data "angles congruent, sides congruent"; a student contributed the warrant "only the side lengths are different"; and the teacher and students jointly contributed the claim "the angles are never going to change in a regular polygon, even if the sides do"). We also include parts of arguments that are not explicitly stated by the teacher or her students and therefore must be inferred from the surrounding context of the classroom and the content of the argument. We call these *implicit* parts, and we indicate them with the clouds in the diagrams (the warrant "if it is true for squares, it is true for polygons" was inferred from the context). This implicit warrant provided the generalization, connecting the data from specific observations of squares to a more general claim about regular polygons. Finally, we include contributions by the teacher that are not components of the arguments but that prompted or responded to parts of the arguments. We represent these by talk bubbles that are connected to the argument components (the teacher's question "Is it a regular polygon?" is an example).

Our adaptation of Toulmin's (1958/2003) model relies on the construct of subarguments to build arguments. Toulmin allowed for subarguments in his model, but he did not elaborate on their status in an episode of argumentation, nor did he illustrate the structure of an argument that included a subargument. He simply stated that they could occur when part of the argument was questioned. Figure 2 illustrates an argument that includes two subarguments. The main claim is "the angles are never going to change in a regular polygon, even if the sides do." That claim is supported by two data that also serve as claims in subarguments: "Both squares are regular polygons" and "You don't have to have a certain size square." Subarguments may arise in two different ways. In the first, they can be preliminary to a claim, with the argument building from one claim into another, so that the argument occurs from left to right in the diagram. In the second, a component, such as data or warrant, is questioned, resulting in it also becoming a claim. In either case, one component serves two purposes: data and claim, or claim and warrant. We label these parts as *Data/Claim* or *Warrant/Claim* and examine their functions both separately from the data, claims, and warrants and combined with the appropriate individual parts. To streamline the diagrams, we insert the contributions that serve two purposes (such as both data and claim) only once.

2.3 Focusing on teacher support

To focus on how teachers support collective argumentation, we pay particular attention to the teacher's direct contributions of argument components, the questions posed to prompt argument components, and the other supportive actions used to facilitate the development of an argument. We differentiate among these different types of support in the same way other researchers have identified and described particular teaching actions used to orchestrate mathematical discussions (e.g., Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Lobato et al., 2005; Staples, 2007). For example, Forman et al. (1998) explored the role of the teacher primarily in the way the teacher revoiced and framed the argument. Lobato et al. (2005) provided a broad perspective and examined the kinds of teaching actions used to support the introduction of new information and mathematical concepts. Among these teaching actions, she and her colleagues included actions in which the teacher contributed aspects of the discussion, asked questions, and encouraged student thinking. Similarly, Staples (2007) characterized the role of the teacher in support of whole-class inquiry as follows: guiding the mathematics, establishing and monitoring a common ground, and supporting students in making contributions (p. 172). In addition, Boaler and Brodie (2004) and Franke et al. (2009) emphasized the role of questions in engaging students in discussions. Although distinct parallels exist among our research and that of others, we believe our framework builds upon and extends the actions that other researchers have identified in a way that specifically characterizes how teachers support collective argumentation as an important subset of their facilitation of classroom discourse.

3 Context

In the following sections, we describe the *teacher support for collective argumentation* framework that arose from our examination of teachers' support for collective argumentation in mathematics classrooms. To contextualize our framework, we describe the study, our data collection, and how we coded our data and conceptualized the framework. We illustrate the framework in the context of one teacher's practice, using episodes of argumentation from two class periods of her ninth-grade class as primary sources for the examples we present. We use those particular lessons because they contain representative examples of unique features of the framework.

3.1 The study

The framework is based on data collected in a study that explored prospective secondary mathematics teachers' beliefs about mathematics, teaching, and proof and how they supported collective argumentation during their student-teaching experiences. Participants in the study were students in two of the first author's teacher preparation courses, one of which was primarily mathematical and the other primarily pedagogical. These courses emphasized, among other things, implementing good tasks and facilitating mathematical discourse (for more information on the courses, see Conner, Edenfield, Gleason, & Ersoz, 2011). The participants were given broad descriptions of the purpose of the study, because we wanted to minimize our influence on what was said in interviews or enacted during student teaching. Our analysis of two participants, Ms. Bell[3] and Ms. Carr, during their subsequent student-teaching experience led to the development of the conceptual framework.

[3] All participant names are pseudonyms.

Ms. Bell and Ms. Carr and the school in which they were placed allowed us to video record a unit of each of their instruction. In the case of each of the other prospective teachers who participated in the first part of the study, either the participant or the school in which he or she was placed did not allow video recording. Thus, our data were limited to these two student teachers. However, Ms. Bell and Ms. Carr were quite different from each other in their beliefs and preferred methods of teaching, allowing us to see different teaching styles in the two classes. Ms. Bell's classes often included small group activities followed by whole class discussions; in contrast, Ms. Carr usually led whole class discussions interspersed with lecture-based notes.

Some readers may wonder about the relevance of basing a framework about teaching on observations of student teachers, arguing that student teachers are only learning to teach and have not developed the more mature teaching skills of experienced teachers. We acknowledge that these are novice teachers who might not enact instruction with the facility of some teachers with more experience. However, while their teaching moves may not have been as mature and well-developed as some experienced teachers, they did enact a wide range of teaching behaviors. They particularly asked a wide range of questions, perhaps because of the emphasis on questioning in their methods course. Thus, we believe that their units of instruction provide a good foundation for our framework of support moves. This contention is further strengthened by our application of the framework to published episodes of collective argumentation as described in Section 3.3.

3.2 Data collection

For their culminating field experience, the student teachers were placed with different mentor teachers in the same rural high school. We conducted approximately 2 weeks of observations in each classroom, capturing an entire mathematics unit. To collect our data, a member of the research team videotaped each of the observed class periods, focusing specifically on the actions of the student teacher, who acted as the classroom teacher during our observations. We also took relevant field notes and collected the tasks and worksheets used during instruction.

3.3 Data analysis

We fully transcribed the video recording of each class, also noting actions such as pointing and other meaningful gestures. The transcripts and video recordings were used as primary data sources, supplemented by the field notes and artifacts. We divided our research group into subgroups to analyze the transcripts for arguments using Toulmin's (1958/2003) model of argumentation. We will illustrate our methods by describing the process of developing the diagram depicted in Fig. 2.

Each subgroup worked on developing diagrams for an entire day's instruction. This was to ensure that the flow of the classroom interaction was uninterrupted analytically and each argument was considered within the context of that day's activity. For example, the diagram in Fig. 2 was situated in a class day devoted to extending the procedure for finding the sum of the interior angles of a polygon to the process of determining the measure of a single interior angle of a regular polygon.

After reading through the transcript, we identified episodes of argumentation and tentatively identified argument components. We purposefully ignored nonmathematical classroom talk and segments of talk that were largely definitional (such as deciding how many sides a nonagon has), and we did not attempt to evaluate the mathematical correctness of the arguments. In focusing on instances where students and teachers made a mathematical claim and provided evidence to support it (our definition of collective argumentation), we were interested in the students' and teacher's understandings rather than our own. We looked for natural breaks in the discourse, which helped to identify episodes of argumentation. In our example, Ms. Bell reiterated the claim and said, "Now how do we find, how do we find these angles in a regular polygon?" This question prompted a shift in focus, signaling a new episode of argumentation.

To identify components, we first looked for the main claim of the argument. The main claim in Fig. 2, *the angles in a regular polygon are never going to change*, *even if the sides do*, was preceded by the linguistic cue, *so*. Because the diagrams are not temporal representations, we then examined the transcript both forwards and backwards to determine the information the students and teacher started with (the data). In our example, Ms. Bell had initiated the discussion by reiterating the class's definition for regular polygons and attributed it to a drawing of a square that she had on the board. She then drew another square and posed the question, "What is different between these two squares?" The two squares were necessary components for the discussion that followed, establishing them as data. Ms. Bell's statement of the definition ensured that the characteristics, *angles congruent*, *sides congruent*, were foregrounded in the reasoning that followed, establishing her statement also as data. It is important to note that neither the drawings of squares nor Ms. Bell's statement about their characteristics was questioned, further supporting their status as data. We then identified other statements (oral, written, and even physical actions) within the episode of argumentation, characterized them as warrants or data/claims, and established the structure of the argument. We noted instances when there were no warrants bridging the data and claim and made inferences based upon what the teacher and students might have meant from the other parts of the argument, including the other warrants explicitly contributed, and contextual details, such as what was written on the board or the teacher's questions. In our example, the students and teacher had established both claims for the squares (*both squares are regular polygons* and *you don't have to have a certain size* [*of a square for it to be a regular polygon*]) and then used those claims as data for the final claim. However, there was no statement that served as a warrant between that data and the final claim, prompting us to infer the warrant, *if it is true for squares then it is true for polygons*.

This pattern held true for all of the diagrams we created during our analysis of the transcripts. After the subgroup had completed the diagrams from the transcript and each member of the research team had read the transcript, the entire research team vetted the proposed diagrams, proposing, debating, and resolving alternative interpretations. This process resulted in 277 final diagrammed episodes of collective argumentation across the two student teachers' units of instruction. As previously described, we coded the diagrams with color and line style to record whether the teacher, the students, or both teacher and students together contributed a given argument component (as in Conner, 2008). Shifting our attention to teacher support, we defined it as any teacher move that elicited or responded to an argument component[4]. We captured questions and other supportive actions by reading through the transcripts and reviewing each class video, carefully watching for gestures and writings on the board that fit our definition of teacher support. These actions were placed within the diagram in talk bubbles connected to the relevant component (see Fig. 2). Diagramming the argument was crucial to identifying a move as supporting argumentation. Not all actions of the teacher were considered support moves. To illustrate how our definition of teacher support

[4] Note that a move did not have to be productive, nor did it have to be part of a productive or mathematically correct argument to be supportive. We interpreted a move as supporting collective argumentation if it elicited or responded to a component of an argument.

guided our analysis, we give an example of a teacher move that did not fit within our definition and was therefore not captured. During the argumentation episode depicted in Fig. 2, after drawing the second square on the board, Ms. Bell instructed the students to "pretend that's a square." We did not capture this comment in a talk bubble because it did not elicit any component and it accompanied, rather than responded to, the data component. As such, Ms. Bell's comment served to establish the figure as a square, which was captured in the data component. Thus, in order to be considered to be supportive of collective argumentation, an action had to elicit or respond to an argument component.

The research group analyzed each type of support separately. We used Toulmin's (1958/2003) model to determine the classifications for the direct contributions (e.g., claim, data, and warrant). Because we were interested specifically in teachers' support for collective argumentation, we chose an inductive approach to our analysis of the teachers' actions beyond direct contributions, resulting in two types: questions and other supportive actions. Other research has developed frameworks for teacher questions in the context of mathematical discussions, but we had no assurance that what we found for collective argumentation would mirror their results. Thus, we did not start with those questions identified for discourse in general, but searched our data for the actions that were specific to supporting collective argumentation. We inductively developed codes for questions and other supportive actions and then collapsed the codes into meaningful categories. For example, in our analysis of the teachers' questions, we carefully reviewed each question that prompted a part of an argument, developed codes for the kinds of questions and definitions for each, and arranged the collection of questions and codes in multiple ways until we developed a more refined and coherent picture of the various kinds of questions asked to support collective argumentation. Then, we looked across the 15 kinds of questions to determine if there were themes existing among them. Through this process, we collapsed the various kinds of questions into five categories of questions used to support collective argumentation. The same process was used to analyze the other supportive actions. After developing our framework, we examined episodes of collective argumentation at various grade levels and with a variety of mathematical content reported in the mathematics education research literature (in particular, we examined arguments reported in Forman et.al, 1998; Knipping, 2003, 2008; Krummheuer, 2007; McClain, 2002; McCrone, 2005; Weber, Maher, Powell, & Lee, 2008; Wood, 1999; Yackel, 2002), and we found that the teachers' actions in these episodes could be categorized with our framework. In addition, when we examined other researchers' characterizations of the actions of the teachers in their work, we found that our framework similarly identified specific teaching actions, and, in some cases, provided more specific insight into the purpose of the move. The three types of support, along with the categories of each and codes within each category, constitute the teacher support for collective argumentation framework[5].

3.4 Description of the focus class periods

We focus our examples and explication of the framework on one of the teachers, Ms. Bell. We observed Ms. Bell teaching one geometry unit (seven 90-min classes) of a ninth-grade accelerated mathematics course. The high school was undergoing a change in its mathematics curriculum, with the ninth-grade course in its second year of implementation of an integrated approach to secondary mathematics. Ms. Bell's instruction usually involved students working in small groups on a mathematical task. She acted as facilitator of these small-group

[5] The three types of support comprise the actual framework. We make the assumption that in order to use the framework most robustly, a user would diagram the episode(s) of argumentation of interest prior to applying the categories of the framework.

explorations, rotating between groups. Small-group work on tasks was followed by whole-class discussions of the task's mathematical content later in the class period or the next day. Ms. Bell's students were motivated and generally considered "good students," but they were not considered "gifted." They had not taken an advanced eighth-grade mathematics course, but this ninth-grade course would allow them to progress through the high school mathematics curriculum at a pace that would end with calculus.

We chose episodes of argumentation from two consecutive classes toward the beginning of the geometry unit. These two classes illustrate most of the kinds of activities in which Ms. Bell and her students engaged. Because of space considerations, we include complete diagrams of only two of the arguments that occurred on these days. As in the diagramming of any episode of argumentation, we present a reconstruction of the final argument; diagrams do not represent the temporal order in which the argument occurred (see also Krummheuer, 1995).

Day 1 began with time for small groups of students to measure the exterior angles of a set of polygons provided on a worksheet and reproduced on the board at the front of the class. Each group of students used a protractor to measure the exterior angles of one polygon and found the sum of those measures, recording their measures and sums on the board. The class concluded that the sum of the exterior angles of any polygon was 360°, and Ms. Bell posed the problem of finding the sum of the measures of the interior angles of any polygon. The students worked on this problem in small groups, first finding the measure of each interior angle of the given polygons, then finding the sums, and finally looking across the sums for a general rule or formula to calculate the sum of the measures of the interior angles of any polygon.

The first episode of collective argumentation that we highlight occurred toward the end of the time during which the small groups of students were looking for a formula to calculate the sum. Ms. Bell approached one of the small groups of students to ascertain their progress. Episode 1 (diagrammed in Fig. 3) began with Ms. Bell asking, "Did you do your table? What

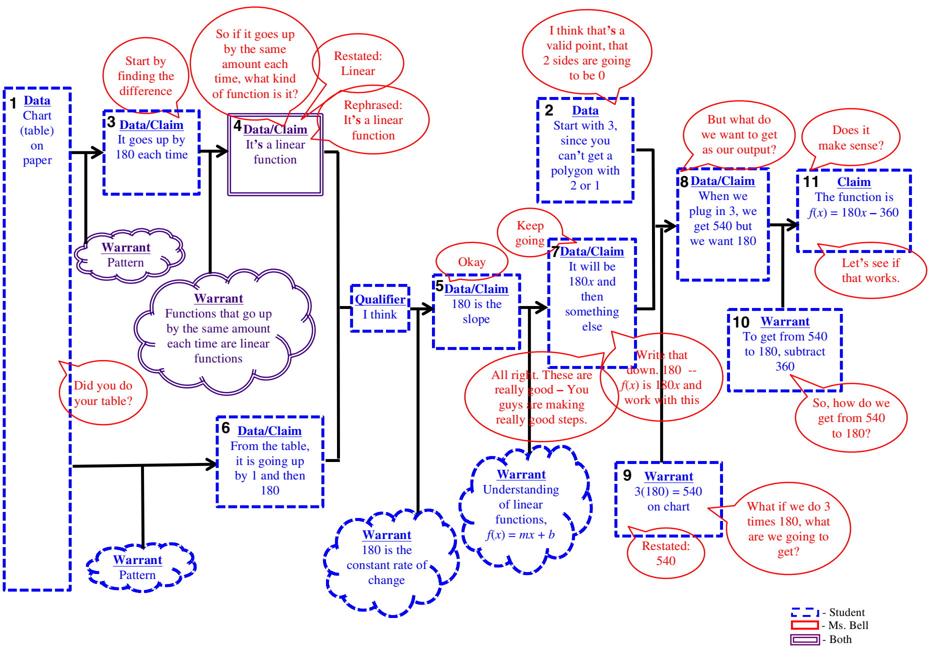


Fig. 3 Diagram of day 1, episode 1

is that?" Martin described the table, and there were a few back-and-forth comments about the table and its contents, which were the number of sides and sums of measures of interior angles of several polygons. (These are the data labeled 1 in the diagram in Fig. 3.) During this time, Martin pointed at the table and said, "Two sides equal zero, three sides … I should probably just ignore that one [referring to two sides]."

Ms. Bell: I think that's a valid point, that two sides is going to be zero.

Adam: And if you have no sides, it's not even an angle. [Data labeled 2.]

Ms. Bell: Mmhm, let's think about it. I think you should start by finding the difference.

Martin: It goes up by 180 each time. [Data/claim labeled 3.]

Ms. Bell: Okay.

Adam: Each one, yeah, and then, uh—.

Ms. Bell: So if it goes up by the same amount each time, what kind of function is it?

Karin: Linear [Data/claim labeled 4.]

Ms. Bell: Linear

Martin: So, it's something

Ms. Bell: It's a linear function.

Karin: I think the slope is going to be 180 [Data/claim labeled 5.] because if you look at the table, it's going up by one and then by 180. [Data/claim labeled 6.]

Ms. Bell: Okay.

Karin: So it's $180x$, and then something to make it so it's not—[Data/claim labeled 7.] When you plug in 3, it doesn't equal like [inaudible] or whatever.

Ms. Bell: All right. These are really good. You guys are making really good steps. Write that down, $180-f(x)$ is $180x$, and work with that. Keep going from there, really good points.

Martin: We've got 540, but we want to get 3—I mean 180. [Data/claim labeled 8.]

Ms. Bell: So, what if we do 3 times 180? What are we going to get?

Martin: 3 times 180, 540. [Warrant labeled 9.]

Ms. Bell: 540, but what do we want to get as our output?

Martin: 180.

Ms. Bell: So, how do we get from 540 to 180?

Karin: I think you've got a defective one though.

Adam: That was the first one.

Martin: Subtract 360. [Warrant labeled 10.]

Ms. Bell: Let's see if that works.

Martin: I got it.

Ms. Bell: Does it make sense?

Martin: Mmhm [yes].

Adam: 180*x* minus 360 [Claim labeled 11.]

Ms. Bell: I don't want you to just write it down.

Karin: That's what I got. I was about to graph it.

Ms. Bell: I want you to understand it. Martin, I want you to do me a favor and explain it to everyone at this table, okay?

After episode 1, Ms. Bell left this small group and worked with a few other groups. At the end of class, Ms. Bell asked two of the groups to share their formulas for the sum of the measures of the interior angles of a polygon. The first group shared two versions of the formula: $f(n)=(n-2)\ 180$ and $f(n)=180n-360$. Their data were the sums of interior angle measures listed on the board, and they used "trial and error" as their warrant for the formula. The second group was the group consisting of Karin, Adam, and Martin. They reported their formula as $f(s)=180s-360$, and their argument essentially mirrored the one constructed from their small-group conversation, emphasizing that this function was linear. This first class period ended with consensus that the sum of the measures of the interior angles of a polygon was $f(s)=180s-360$, where s is the number of sides of the polygon. This episode is presented as an example of the collective argumentation in Ms. Bell's class. Clearly, the students were noticing patterns and making conjectures about mathematical relationships. In so doing, they were engaging in inductive reasoning. We do not claim that this episode in any way demonstrates the deductive reasoning necessary for mathematical proof. Rather, the students were engaged in inductive reasoning appropriate for their task, and the teacher supported their reasoning in constructive ways.

Day 2 began with the students finding the sum of the measures of the angles of a 12-sided polygon as a quick review of the previous day's work. Ms. Bell then asked the students to think about another way to find the sum of the interior angles of any polygon by subdividing several polygons into the least number of triangles. Together, she and the class connected the formula from the end of day 1 to the process of subdividing polygons into triangles. After making that connection, Ms. Bell introduced the term *regular polygon* and asked the students to think about what *regular* meant in everyday language, eventually connecting their intuitive notions to a class-constructed definition of "all sides congruent, all angles congruent." From this definition, the class came to the understanding that all regular quadrilaterals will have congruent angles, all regular pentagons will have congruent angles, and so on, even though the side lengths may change.

At this point in the lesson, a student made a comment that led to an investigation by the class that Ms. Bell had not anticipated. Karin asked a question in which she compared a rhombus to a square and asked if anyone could predict what the angle measures in a rhombus would be, acknowledging that they would be different from 90°. Ms. Bell rephrased Karin's question for the class, which is the beginning of the following excerpt. This highlighted episode, episode 2 (diagrammed in Fig. 4), illustrates the interactions of Ms. Bell and her students when they were investigating a question that had been posed by a student.

> Ms. Bell: So Karin, you're saying, we know that the interior angle sum is 360. [Data labeled 1 in the diagram.] We know that these angles are the same and these angles are the same [points at opposite angles in the rhombus drawn on the board]. [Data labeled 2 in the diagram.] And we know that these angles [points at the same side angles] sum to what?
>
> Micah: 180 [Data labeled 3 in the diagram.]
>
> Ms. Bell: 180. Do we know anything else? Can we use that information to find what the interior angle of any rhombus is going to be? [8 s pause]

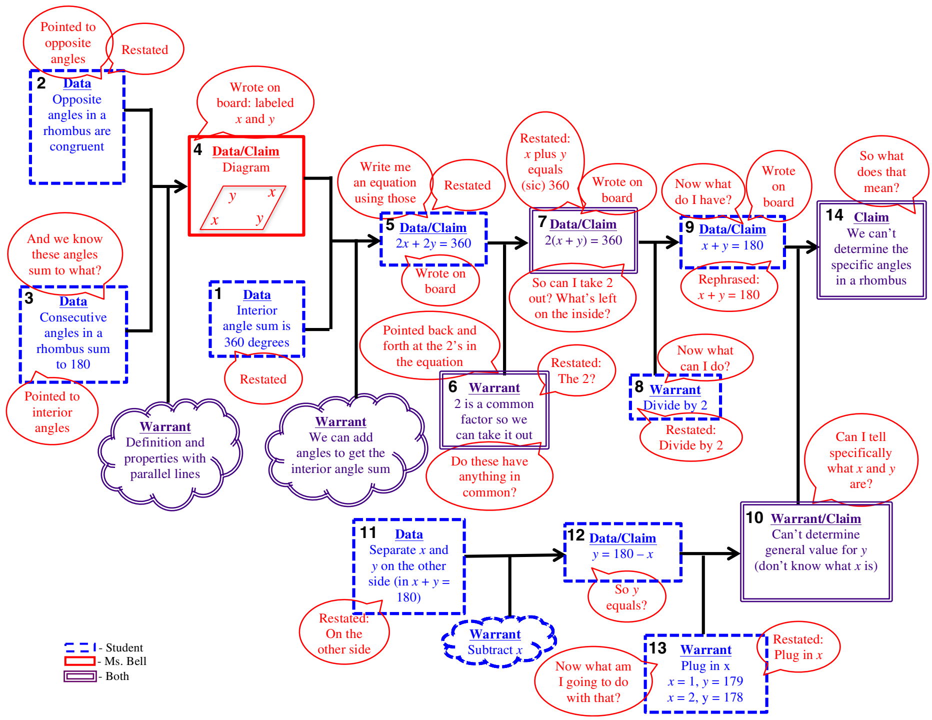


Fig. 4 Diagram of day 2, episode 2

Angela: You should.

Ms. Bell: You should? Should be able to, hmm. [5 s pause] I don't know. I'm asking y'all, I really don't know.

Micah: Let's test it.

Ms. Bell: You want to test? You want to draw any rhombus on your paper? You guys want to do that? You might need to use a straight edge. [Students work on drawing rhombi for 27 s.] So these angles, we could call them x [labels two of the congruent angles x in the rhombus on the board.] Call these two angles y [writes y in the other two angles in the rhombus.] [Data/claim labeled 4 in the diagram.] So, write me an equation using those.

Martin: $2x+2y=360$. [Data/claim labeled 5 in the diagram.]

Ms. Bell: $2x+2y=360$ [writes $2x+2y=360$]. Now, what can I do?

Travis: You could do $2x$ squared.

Ms. Bell: $2x$ squared?

Travis: Plus $2xy$ equals—.

Adam: No, you could just do—.

Travis: Never mind, I was thinking like that.

Ms. Bell: Area?

Adam: a squared.

Travis: a squared plus b squared equals c squared.

Ms. Bell: Oh, oh, okay, interesting.[6] Do these have anything in common? [Ms. Bell indicates $2x$ and $2y$]

Micah: I got a good one.

Karin: Two [Together with Ms. Bell's contribution, the warrant labeled 6 in the diagram.]

Ms. Bell: The two? So can I take a two out? What's left on the inside?

Karin: x plus y. [Together with Ms. Bell's contribution, the data/claim labeled 7 in the diagram.]

[6] This statement and the preceding eight lines did not add to the argument but are included here in the transcript for the sake of completeness and to illustrate that when diagramming arguments, there are decisions that must be made concerning what actually contributed to the argument.

Ms. Bell: x plus y equals (sic) 360 [writes $2(x+y) = 360$]. Now, what can I do?

Karin: Divide by two [The warrant labeled 8 in the diagram.]

Ms. Bell: Divide by two. Okay. Now what do I have?

Micah: Equals 180. [The data/claim labeled 9 in the diagram.]

Ms. Bell: x plus y equals 180 [writes $x+y=180$]. But can I tell what x and y specifically are?

Adam: Not so. [Together with Ms. Bell's contribution and Micah's later contribution, part of warrant labeled 10 in the diagram.][7]

Martin: You could separate them and get x on the other side. [Data labeled 11 in the diagram.]

Ms. Bell: x on the other [side], so y equals?

Martin: 180 minus x. That's what I got. [Data/claim labeled 12 in the diagram.]

Ms. Bell: Now, what am I going to do with that?

Martin: You're going to plug in x. [Part of warrant labeled 13 in the diagram.]

Ms. Bell: Plug in x.

Micah: But we don't know x. [Part of warrant labeled 10 in the diagram.]

Ms. Bell: So what does that mean? How many—I can plug in values for x

Martin: x equals 1; y equals 179. [Part of warrant labeled 13 in the diagram.]

Ms. Bell: Umhmm.

Martin: If x equals 2, y equals 178 [Part of warrant labeled 13 in the diagram.]

Ms. Bell: So, I guess what we're saying is we really can't, we can't tell, I don't think all rhombuses have the same measures for x and y. [Claim labeled 14 in the diagram.] That's what this is saying, right? Okay. We just got really off task, right there. But that was interesting.

Martin: But I learned something.

After this digression, Ms. Bell again engaged the whole class in investigating the previous question of finding the measure of one angle in a regular polygon. The students concluded in

[7] Although this statement was made by a student, Ms. Bell's preceding question suggested only one possible answer. Therefore, the component was attributed to both student and teacher.

specific cases that to find the measure of one angle of a regular *n*-gon, they should divide the sum of the measures, found with their previous formula, by *n*. From this conclusion, Ms. Bell asked if they could find the measure of one exterior angle of a regular polygon. The students concluded that they could, using the examples of a regular hexagon and a regular pentagon. For the rest of the class period, Ms. Bell set small groups of students to the task of proving that the sum of the measures of the exterior angles of a polygon is 360°.

4 How teachers support collective argumentation

The teachers in our study used three types of support for collective argumentation: directly contributing argument components, asking questions that elicited parts of arguments, and using other supportive actions. These three types of support make up the teacher support for collective argumentation framework (see Table 1) and are described in subsequent sections. Even though each part of the framework can be used separately and analysis of a teacher's practice using each type of support can yield interesting results, we believe the power of the framework can be seen when the three types of support are analyzed together in the context of collective argumentation. We assume that to use the framework to analyze the particularly mathematical aspects of classroom discourse, a researcher would diagram the argumentation prior to applying the framework.

4.1 Direct contributions to arguments

One of the most obvious ways in which teachers may support collective argumentation is by providing parts of arguments, which we call *direct contributions*. In our analysis, we distinguish between parts contributed by the teacher, by students, or collaboratively by the teacher and students together. By direct contributions, we mean those components that were contributed by the teacher without obvious contribution by students. These specific parts of arguments could be contributed by making a statement verbally, by writing something on the board, or by presenting a written task. We considered parts of tasks posed by the teacher as part of the teacher's direct contributions to arguments when these parts served as data in arguments.

We used Toulmin's (1958/2003) constructs to classify the teachers' direct contributions of argument components. In addition, we analyzed contributions of components that served as both claims and data (data/claims), both claims and warrants (warrant/claims), and both data and warrants (data/warrants). To illustrate a teacher's direct contributions, we refer to episode 2 (Fig. 4) in which Ms. Bell directly contributed a data/claim to the argument by drawing a diagram on the board. Another illustration can be found in the argument diagrammed in Fig. 2, in which Ms. Bell directly contributed two data and one warrant by drawing figures and stating facts about them.

4.2 Asking questions

Our analysis generated 15 different codes for the kinds of questions used to support collective argumentation. By *question*, we mean a request for action or information, not simply an interrogative sentence. Thus, rhetorical questions were not included in this type of support for collective argumentation, whereas statements requesting an action were included. This methodological decision is consistent with other characterizations of questions in the literature (e.g., Boaler & Brodie, 2004). Our analysis of teachers' questions was done from the viewpoint of the teacher, so in our analysis, we inferred the teacher's intent in asking the question rather than

Table 1 Teacher support for collective argumentation framework

Direct contributions		Questions		Other supportive actions	
Claims	Statements whose validity is being established	*Requesting a factual answer*	Asks students to provide a mathematical fact	*Directing*	Actions that serve to direct the students' attention and/or the argument
Data	Statements provided as support for the claims	*Requesting a method*	Asks students to demonstrate or describe how they did or would do something	*Promoting*	Actions that serve to promote mathematical exploration
Warrants	Statements that connect data with claims	*Requesting an idea*	Asks students to compare, coordinate, or generate mathematical ideas	*Evaluating*	Actions that center on the correctness of the mathematics
Rebuttals	Statements describing circumstances under which the warrants would not be valid	*Requesting elaboration*	Asks students to elaborate on some idea, statement, or diagram	*Informing*	Actions that provide information for the argument
Qualifiers	Statements describing the certainty with which a claim is made	*Requesting evaluation*	Asks students to evaluate a mathematical idea	*Repeating*	Actions that repeat what has been or is being stated
Backings	Usually unstated, dealing with the field in which the argument occurs				

classifying questions by the types of responses they elicited from students. We organized these kinds of questions thematically. This process allowed us to characterize the various codes for the kinds of questions into five general categories: requesting a factual answer, requesting an idea, requesting a method, requesting elaboration, and requesting evaluation (see Table 2). It is important to emphasize that our presentation of categories of questions in the framework is not hierarchical, nor do we value certain categories over others. We present these categories of questions as a means to describe the ways in which teachers might support collective argumentation in their classrooms. Throughout the remainder of this section, we explicate the categories in our framework and illustrate them with examples from Ms. Bell's teaching.

To support collective argumentation, at times, Ms. Bell asked students questions *requesting a factual answer*. In this category, Ms. Bell asked various kinds of questions that prompted her students to contribute facts by providing an immediate answer or by performing a simple mathematical action. As examples, there were times when Ms. Bell asked her students to provide a previous result, the answer to a homework problem or an idea generated in their small groups. Sometimes the student's contribution of a previous result was followed by a longer discussion of mathematical ideas. In day 2, Ms. Bell asked, "What is the sum of the exterior angles of any polygon?" Ms. Bell's question requested students to contribute a claim previously established in day 1, the sum of the exterior angles of any polygon is 360 °.

There were times when Ms. Bell asked students to demonstrate a method or describe a method used to arrive at a specific answer. We categorized these kinds of questions as those *requesting a method*. For instance, when asking for a demonstration, Ms. Bell typically asked a student to share his or her method with the class by providing work on the front board. In day

Table 2 Categories of questions in support of collective argumentation

Category of question	Kind of question with example
Requesting a factual answer: Asks students to provide a mathematical fact	*Calculation*: "10 times 180 is how many degrees?"
	Identification: "Which angles are congruent [in the parallelogram]?"
	Previous result: "What is the sum of the exterior angles of any polygon?"
	Recall: "A pentagon has how many sides?"
	Term: "What is it called when angles sum to 180?"
Requesting an idea: Asks students to compare, coordinate, or generate mathematical ideas	*Comparison*: "Do these have anything in common? [Ms. Bell indicates $2x$ and $2y$]"
	Conjecture: "If you know how to find a specific interior angle of a regular polygon, can I find what the exterior angles are all going to be equal to?"
	Construct result: "So can I take a 2 out? What's left on the inside?"
Requesting a method: Asks students to demonstrate or describe how they did something	*Demonstrate a method*: "So, will one of y'all go up and show next? So, Travis, tell us what you did."
	Describe a method: "How did you figure that out?"
Requesting elaboration: Asks students to elaborate on some idea, statement, or diagram	*Explanation*: "Can you explain it a different way?"
	Interpretation: "So if it goes up by the same amount each time, what kind of function is it?"
	Justification: "Why?" or "Why doesn't that work?"
Requesting evaluation: Asks students to evaluate a mathematical idea	*Consensus*: "Do you guys agree with that?"
	Reconsider: "Really? Any polygon, huh?"

1, after working in small groups, Ms. Bell asked a student to demonstrate the formula he derived. She asked, “So, will one of y’all go up and show next? So, Travis, tell us what you did.” Travis responded to Ms. Bell’s question by sharing his method with the class at the front board. Using either kind of question in this category, it was possible for Ms. Bell to solicit multiple responses from students in order to generate a variety of solution methods and further the class discussion.

Across many episodes of collective argumentation, Ms. Bell posed specific questions *requesting a mathematical idea*, asking students to compare, coordinate, or generate mathematical ideas. For example, in episode 2, Ms. Bell asked a sequence of questions that requested mathematical ideas. First, she asked her students to make a comparison between $2x$ and $2y$, “Do these have anything in common?” Responding to this question, her students observed that 2 was common in each term; and, in doing so, they contributed part of a warrant in the given episode. To continue the discussion, Ms. Bell asked a second question. She expected students to coordinate mathematical ideas to construct a mathematical result. She asked, “So can I take a 2 out? What’s left on the inside?” With this kind of question, Ms. Bell expected students to construct a mathematical result by factoring out a 2 from the sum $2x+2y$. By asking questions in this category, she expected students to provide a mathematical idea, which contributed to the development of the argument.

There were times when Ms. Bell posed questions *requesting elaboration*. Questions in this category prompted students to elaborate on their thinking regarding a particular mathematical idea. This elaboration took the form of asking students to make an interpretation, to contribute an explanation, or to provide justification. As an example, in day 1, Travis demonstrated his method for finding the formula for the sum of the measures of the interior angles of a triangle for the class. Ms. Bell asked him to provide an explanation of the work he was presenting on the board, saying, “What is this you are writing on the board?” By asking this question, she expected him to expand upon his written work by providing additional description or clarification. In this category, Ms. Bell’s requests for elaboration were often addressed to the entire class, and the elaboration of ideas typically involved students supporting their claims with data and warrants.

There were times when Ms. Bell asked questions requiring students to either agree or disagree with a specific mathematical idea. We categorized these kinds of questions as those *requesting evaluation*. Sometimes, Ms. Bell asked questions of consensus, to determine if there was a general agreement among the students about a mathematical claim, whereas others of Ms. Bell’s questions in this category encouraged students to reconsider a mathematical idea, implying the idea was not mathematically correct or complete. One particular example occurred on day 2 when Ms. Bell asked her students to reconsider a student’s claim stating that it was possible to determine the number of sides of any polygon by dividing 360° by the measure of an exterior angle of the polygon. Ms. Bell asked, “Really? Any polygon, huh?” Following this question, students clarified their claim to include “regular” to describe the kind of polygon.

4.3 Other supportive actions

In our analysis, we noticed teachers supported collective argumentation with actions that were not questions or direct contributions to the argument. Our analysis generated 13 codes for the different kinds of other supportive actions. We collapsed the codes into five meaningful categories: directing, promoting, evaluating, informing, and repeating actions (see Table 3). We present these categories of other supportive actions to assist in describing the ways in which teachers might support collective argumentation in their classrooms. In this section, we illustrate each category with an example from Ms. Bell’s teaching.

Table 3 Categories of other supportive actions in support of collective argumentation

Category of other supportive action	Kind of supportive action with example or explanation
Directing: Actions that serve to focus the students' attention and/or the argument	*Highlights* a particular part of the activity, diagram, or procedure
	Hints: "I think you should start by finding the difference."
	Refocuses students' attention on an important aspect of the activity
Promoting: Actions that serve to support mathematical exploration	*Encourages*: "These are really good. You guys are making really good steps."
	Suggests: "You might have to try something else."
Evaluating: Actions that center on the correctness of the mathematics	*Corrects* a student's incorrect statement
	Validates: "Now, I like that definition."
	Verifies the correctness of a contribution to the class
Informing: Actions that provide information for the argument	*Clarifies* statements with descriptions or gestures
	Expands on a student's response to describe the student's contribution more fully
	Summarizes the main points in a student's contribution
Repeating: Actions that repeat what has been or is being stated	*Displays* a student's or teacher's contribution on the board
	Restates a student's contribution

To support collective argumentation, there were times when Ms. Bell used *directing* actions to focus students' attention to a specific mathematical point or to steer an argument in a specific direction. For example, in episode 1, Ms. Bell provided a hint to some students working in a small group. Ms. Bell told the group, "I think you should start by finding the difference." With this hint, Ms. Bell focused the argument by directly telling the students what to do next in order to progress through the mathematical task. In this category, these actions directed the students' thinking toward important mathematical aspects in the argument.

Ms. Bell used *promoting* actions to support students' exploration of mathematics or their contributions of parts of arguments. Unlike the directing actions, the actions in this category tended to allow the argument to move in many possible directions, by encouraging or suggesting that students continue their reasoning, activity, and/or argument. For example, in episode 1, Ms. Bell encouraged the small group of students, "These are really good. You guys are making really good steps." In this example, Ms. Bell encouraged students to continue in their thinking. She did not, however, guide the argument in a specific direction.

Ms. Bell used *evaluating* actions to validate a student's contribution to an argument, to verify the correctness of a student's statement, or to correct a student's statement. For example, Ms. Bell often validated a student's contribution by indicating the contribution was mathematically appropriate or correct. For example, after the class had developed their definition of a regular polygon on day 2, Ms. Bell stated, "Now, I like that definition." She validated the definition without explicitly promoting students to continue their reasoning. These evaluative actions consistently communicated the soundness of students' claims or reasoning to the class.

There were times when Ms. Bell used *informing* actions to assist students in developing mathematical understanding and problem solving strategies. The information Ms. Bell provided most often took the form of direct contributions of argument components. There were times, however, when Ms. Bell provided information that did not directly contribute a component in an argument. These informative statements supported the argument in other ways, by clarifying statements or summarizing or expanding on what a student said. For example, in episode 2, Ms. Bell pointed at the opposite angles in the rhombus to clarify the angles to which a student referred (see data labeled 2 in episode 2).

Many times, Ms. Bell supported her students' contributions by announcing to the class what a student said. These *repeating* actions generally took the form of restating what a student said or displaying it on the board without adding or subtracting information. We differentiated repeating actions from informing actions (such as summarizing) by examining whether the teacher changed the contributed information in some way. In this category, Ms. Bell's main goal was simply to ensure the class heard the information. For example, in episode 1, Ms. Bell both restated Martin's equation, "$2x$ plus $2y$ equals 360" and displayed it on the board using the appropriate symbols. With these actions, she ensured that the class both heard and saw his contribution.

5 Contributions of the framework

We propose this framework as a device that allows one to examine conversations in mathematics classrooms in a particularly mathematical way. In the following paragraphs, we describe how we can use this framework within other frameworks, such as Hufferd-Ackles et al.'s (2004) math talk framework or Rasmussen and Stephan's (2008) method for documenting collective activity to look more specifically at which actions of the teacher are productive. We examine the connections between our framework and how teachers' actions in support of collective argumentation are described in the literature. We also examine how our framework allows a specific focus on warrants and the ways teachers provide or support their students in providing warrants. We suggest some ways in which using our framework can focus one's attention on the more mathematical aspects of classroom activity. Finally, we describe some limitations of our framework with respect to its source and applicability.

5.1 Connections to frameworks for classroom activity

Other frameworks, such as Hufferd-Ackles et al.'s (2004) math-talk framework or Stein et al.'s (2008) five practices, have provided useful ways to examine the growth of mathematical communities and orchestrate discourse. Our framework, whether used in conjunction with one of these frameworks or separately, allows one to look more closely at specific mathematical aspects of the conversation. It allows us to view classroom behavior on a more micro level, categorizing individual actions of the teacher before looking across these to examine patterns.

As an example, we examined Ms. Bell's classroom community through the lens of Hufferd-Ackles et al.'s (2004) math-talk framework. Although we did not have data to look at the growth of the community over time, we were able to place Ms. Bell's class on the math-talk continuum for each of the four categories. Briefly, we placed the interactions in Ms. Bell's class at levels 2 or 3 in each of the areas. However, our framework allowed us to provide a finer-grained analysis of the mathematical conversations. For instance, in Hufferd-Ackles et al.'s "source of mathematical ideas" category, our framework provided detail that suggests what kinds of mathematical ideas students learned because the class was at level 3, at least for the class periods under close scrutiny in this paper.

Hufferd-Ackles et al. (2004) give the following description of level 3 of the math-talk framework: "Teacher allows for interruptions from students during her explanations; she lets students explain and 'own' new strategies…. Teacher uses student ideas and methods as the basis for lessons or miniextensions" (p. 90). Our description of Ms. Bell's classes includes an example of Ms. Bell's use of a student's question as impetus for an unanticipated investigation when she rephrased Karin's question, using it and Micah's request to initiate an investigation of angle measures in a rhombus. In so doing, she leveraged many of her students' ideas to support their eventual conclusion that there was not one solution to one equation containing two unknown quantities and thus the angles in a rhombus could not be predicted in the same way as the angles in a square. Although the conclusion about a rhombus was related to the mathematical ideas being discussed on that day, the conclusion about the number of solutions (or lack of a unique solution) to an equation with two unknowns was not. Ms. Bell's introduction of this important mathematical idea in an otherwise unrelated discussion was evident through our analysis of her questions and the warrants provided in her class. Thus, although the math-talk framework might flag this episode as important, our framework specifies why it is mathematically important: by direct contributions (e.g., providing a diagram of a rhombus on the board), other support (e.g., labeling the measures of the angles in the rhombus with variables), and her questions (e.g., "Can I tell what x and y specifically are?"), Ms. Bell supported her students in using appropriate disciplinary reasoning to draw conclusions about the geometric content under consideration by using a concept from algebra and functions.

The teacher support for collective argumentation framework goes beyond frameworks currently in use to allow examination of how teachers impact the development of mathematics in classrooms and how teachers support the reasoning of students. Rasmussen and Stephan (2008) have articulated the usefulness of collective argumentation in documenting the progression of collective knowledge in classrooms by recording how specific ideas progress from being claims requiring data and warrants to being used as data or warrants in support of other claims. Our framework allows examination of how the teacher supports students in understanding how and when claims that have been previously established can be used in support of subsequent claims. As a simple example, in several of the day 2 arguments, Ms. Bell asked questions that prompted students to use the formula they had established on day 1 directly in finding the sum of the measures of the interior angles of particular polygons and also in finding a general method for finding the measure of one interior angle of a regular n-gon. The teacher can also support the students' use of established claims by other supportive actions such as validating the contribution of such claims as data or warrants or displaying relevant parts of the contribution. In some cases, the teacher might model the use of previously established claims by directly contributing them within the argumentation.

5.2 Connections to other descriptions of teacher moves

Within the mathematics education literature on collective argumentation, several authors have described the actions of the teacher when facilitating argumentation. Many of those actions fit within our framework of support for collective argumentation. Our framework serves to consolidate the variety of moves described by authors, building on their observations of specific moves. However, many researchers report teacher actions observed at a different grain size, examining things such as establishing norms (e.g., Weber et al., 2008) or focusing or funneling (Wood, 1998). Our framework allows identification of the individual moves that may go into such larger activities, and patterns of the individual moves may allow an investigation of those larger activities, but we have not yet used our framework to examine those larger activities. We have, however, examined the moves of teachers described in reports of collective argumentation and connected them to our framework. Table 4 provides a summary of some of the moves specifically identified in a subset of the literature that reported teachers' actions with respect to collective argumentation and their relation to our proposed framework of support moves.

Examining patterns within individual teacher moves may give insight into a larger activity, even outside an explicit examination of collective argumentation. Herbel-Eisenmann and

Table 4 Selected teacher moves from literature reporting teachers' actions with respect to collective argumentation

Author(s)	Identified teacher moves	Category of support for collective argumentation framework
Forman et al. (1998)	Teacher asks a volunteer to "give and justify his or her answers" (p. 535)	Questions, requesting elaboration: justification
Forman et al. (1998)	"The teacher revoices Larry's grounds via repetition and expansion" (p. 536)	Other supportive actions, informing: expands
Forman et al. (1998)	"The teacher summarizes the two arguments" (p. 540)	Other supportive actions, informing: summarizes
Knipping (2003)	"The teacher encourages the students to formulate a conjecture…" (p. 4)	Questions, requesting an idea: conjecture
Knipping (2003)	"The teacher expresses feelings of appreciation when the student comes up with his conjecture" (p. 4)	Other supportive actions, promoting: encourages
Knipping (2003)	Teacher "encourages the class to confirm or reject this conjecture" (p. 4)	Questions, requesting evaluation: consensus
Wood (1999)	Teacher asked a student "to clarify for the others why she [a student] was questioning…" (p. 181)	Questions, requesting elaboration: explanation
Wood (1999)	Teacher "indicated that he [a student] was to give reasons for his thinking" (p. 181)	Questions, requesting elaboration: justification
Yackel (2002)	Teacher "capitalized on students' contributions" (p. 430)	Other supportive actions, repeating: restating
Yackel (2002)	"The teacher drew a diagram on the chalkboard" (p. 432)	Other supportive actions, repeating: display
Yackel (2002)	Teacher asked a student "to explain why he added 8 and 8" (p. 436). "She could facilitate the discussion best by directly calling for a warrant herself" (p. 436).	Questions, requesting elaboration: explanation

Breyfogle (2005) suggest that the patterns of questions posed by teachers provide insights into the types of interactions teachers have with their students. These interactions include *funneling* and *focusing* interactions. Wood (1998) indicates that funneling interactions occur when a teacher asks a series of questions that direct students through a procedure or to a desired solution such that the teacher is doing the majority of the mathematical thinking and the students are simply providing answers that may seem unconnected. Focusing interactions occur when a teacher listens to students' responses and asks questions that direct them based on what the students are thinking rather than their own solution method (Wood, 1998). Although these interactions are centered on the patterns of questions, the actions of the teacher play an important role. In her examples of these interactions, Wood also provides a description of the teacher's actions and states how these actions support the specific type of interaction. Using the teacher support for collective argumentation framework, researchers can examine the kinds of questions and other supportive actions that compose patterns of funneling and focusing interactions and the structures of the arguments in which these interactions take place.

Forman et al. (1998) described several teacher actions under the category of revoicing. These actions involve moves in two different categories of other supportive actions: informing and repeating. Chapin, O'Conner, and Anderson (2003) include revoicing as an essential math-talk move. In their discussion, they focus on the importance of clarifying students' contributions to the discussion and making the contributions of individual students understandable to the other members of the class. This talk move is similar to our other supportive action of informing. However, from our analysis of practice, we found it useful to distinguish among moves that clarify a student's contribution, summarize a student's contribution, or expand upon a student's contribution. Examining how and when a teacher clarifies, summarizes, or expands a student's contribution, whether each argument was mathematically productive, and who dominated the contributions to and direction of the argument may allow us to make further inferences about when it is productive or appropriate to engage in the wide range of behaviors captured by the framework.

5.3 Focus on warrants

The teacher support for collective argumentation framework allows an explicit focus on the warrants provided in a class as an indication of the reasoning (see also Conner, 2012) that is being made public. This framework allows one to examine who contributes the reasoning in the class and what the teacher does to prompt and respond to the reasoning that is contributed. The teacher's direct contributions of warrants provide part of the picture of the reasoning that he or she values, but it is also important to examine when he or she does not contribute warrants and whether in those instances he or she prompts a student to provide a warrant, a student provides a warrant without prompting, or the reasoning is not explicated because the warrant is implicit. Sometimes when teachers directly contribute parts of arguments, such as warrants, they may take over the argument, making the argument more a construction of the teacher than one that is collectively established. However, other teacher contributions serve to make parts of arguments that would otherwise be left implicit apparent for students or to emphasize the importance of having and presenting reasons for claims. Because reasoning is a pivotal part of mathematics, we believe this feature is an important part of the usefulness of our framework.

One example of the information provided by looking at the warrants in arguments comes from our analysis of day 1 of Ms. Bell's class. In the small-group argument highlighted on day 1, the group left implicit the warrant that connects the observation that "it goes up by 180 each time" to "it's a linear function." Ms. Bell implied the warrant in her question that prompted the data/claim "it's a linear function," but did not make it explicit. However, when this small group presented their argument to the entire class later in that class period, they used essentially the same data/claims as before (using "interior angles sum changes by 180 each time" and "it is linear"), but Ms. Bell directly contributed the warrant, saying, "If it changes by the same amount each time when you are going up by one, it is going to be a linear function." In so doing, she explicated the group's reasoning to the larger class. By not allowing the warrant to be left implicit, Ms. Bell contributed a valid mathematical reason that connected the pattern to the relevant mathematical concept. By contributing the warrant, she modeled the activity of explicating reasoning and made sure that the reasoning was made available to the collective.

5.4 Power and limitations of the framework

One of the aspects of the framework that makes it specifically mathematical is its use in the context of collective argumentation. Although we believe that it is still useful to look at questions and other supportive actions without diagramming the argumentation that occurs in a classroom, it is when the questions and other supportive actions are examined in the context of the claims, data, and warrants of an argument that the most mathematical aspects of the classroom conversations come to light. In particular, questions and other supportive actions are mathematically important when they prompt and reinforce students' contributions of appropriate mathematical claims, data, and warrants. The teacher's direct contributions, or withholding of contributions, or prompting of students to provide contributions to particular parts of arguments can give insight into what the teacher values or privileges in a mathematical conversation. A teacher could use the framework to examine his or her practice without diagramming, but for a robust analysis, a researcher would need to diagram the arguments as we have illustrated in this paper so that the supportive actions can be connected to the relevant parts of the framework and conclusions drawn accordingly.

Part of the power of the framework in conjunction with the diagramming of arguments is that a diagram functions as a sieve. Placing the parts of the argument in the diagram focuses the researcher on the specific mathematical statements and actions that took place in the class and excludes the other classroom activities. From the diagrammed components and the accompanying questions and other supportive actions, the actual mathematical ideas that were foregrounded in the class become apparent. Additionally, the implicit parts of the diagram highlight the mathematical ideas that may have been present but were not stated or necessarily accepted by the class. Using the framework, a researcher can then examine how the various questions and other supportive actions functioned within the mathematical conversation to highlight or fail to highlight important mathematical ideas.

While our framework does allow an in-depth analysis of the argumentation in a classroom, it does not provide a value judgment as to the productivity of the argumentation observed or the usefulness of any particular contribution of a teacher or student. Those judgments are left to the users of the framework. We specifically did not limit our analysis to those episodes of argumentation that we judged to be particularly productive, because we wanted our framework to be applicable to the wide range of argumentation currently present in classrooms. This means that some arguments may be incomplete or mathematically incorrect. It means that sometimes the identified support for collective argumentation may actually involve the teacher "taking over" the argument or providing components of arguments that would have more productively been left to the student. What the framework does in these cases is allow the user to identify what the teacher and students did and then to draw conclusions about whether that would be productive based on the context and his or her mathematical knowledge. An open question remains concerning what patterns of support moves will support *productive* argumentation. More research is needed to determine the characteristics of productive argumentation and how teachers can support such argumentation.

6 Conclusion

Facilitating productive mathematical discussions can be a difficult endeavor for many teachers. To overcome this difficulty, various researchers have examined practices that assist teachers in orchestrating these discussions (Hufferd-Ackles et al., 2004; Staples, 2007; Stein et al., 2008). The teacher actions they propose, such as establishing communities, negotiating responsibility for actions, and paying attention to student solutions, are very broad. Our framework provides a finer-grained perspective on the different ways teachers can support collective argumentation and provides teachers with additional ways to think about their actions during these types of discussions.

The teacher support for collective argumentation framework can be used to inform teacher education and professional development activities. Mathematics teacher educators can use the framework with prospective teachers to engage in a dialog about issues of practice. Prospective teachers may reflect upon the types of support as they begin to construct their identities as mathematics teachers. In addition, understanding the different types of supportive actions could provide practicing teachers with meaningful constructs from which they may examine and reflect upon their practice. For example, knowledge of the different argument components may prompt a practicing teacher to examine the extent to which students contribute components of arguments. This examination of practice may lead the teacher to reflect upon how the other types of support may be used to prompt different and better contributions from students.

The framework allows researchers to compare teachers' practices or to conduct a longitudinal examination of one teacher's practice over time. For instance, perhaps a teacher is engaged in a professional development program in which he or she is seeking more student engagement in mathematical practices such as reasoning. One way to examine the changes in his or her practice would be to diagram selected instances of argumentation and to focus on the questions being asked, the kinds of supportive actions employed, and the results of these actions in terms of student and teacher contributions of warrants and components of subarguments. The framework allows an examination of the teacher's supportive role in engaging students in making and defending mathematical claims.

To continue to address the call by Yackel (2002), the framework allows researchers to generate cases of teacher support for collective argumentation. Shulman (1986) described case-based research literature as a powerful educational tool, because cases can be used to "illuminate both the practical and theoretical" (p. 11). These cases of teacher support for collective argumentation, developed with the common language provided by our framework, could move researchers closer to defining what makes collective argumentation productive. Comparisons across cases of teachers' support would allow for better understanding of how different combinations of support assist students in making their reasoning more explicit for others. These comparisons could also give insight into how teachers can incorporate the students' reasoning into the structure of the argument in productive ways.

Mathematics educators can use this framework to monitor their students' progress toward engaging in deductive reasoning and eventually toward proof as they examine who is providing what kinds of reasoning in the arguments they are making. This framework allows the close examination of the development of sociomathematical norms, particularly in the areas of justification and explanation, as mathematics educators can trace how students contribute parts of arguments, particularly warrants, with and without teachers' support. Arguably, as students become more mathematically autonomous, teachers should be able to provide fewer direct contributions and expect their students to provide parts of arguments, particularly warrants, without needing to be specifically asked for them. Using this framework, mathematics educators can trace the amounts and kinds of teacher support for collective argumentation over time with the goal of characterizing the sociomathematical norms of the class and the disposition of the students with respect to engaging in reasoning and proof.

The teacher support for collective argumentation framework assisted us in making sense of the complexities involved in the ways teachers support collective argumentation. With this framework, we can examine the specifically mathematical aspects of classroom discussions and trace the reasoning that occurs within the class. The types of support we have outlined and the interactions between them in the context of collective argumentation allow us to examine aspects of teachers' activity that may be important in the development of appropriate disciplinary practices. Examining these types of support and their influence on developing disciplinary practices may lead to a more definitive characterization of productive collective argumentation.

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