Justification as a teaching and learning practice: Its (potential) multifaceted role in middle grades mathematics classrooms

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\noindent \textbf{A R T I C L E \ I N F O}

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\noindent \textbf{A B S T R A C T}

Justification is a core mathematics practice. Although the purposes of justification in the mathematician community have been studied extensively, we know relatively little about its role in K-12 classrooms. This paper documents the range of purposes identified by 12 middle grades teachers who were working actively to incorporate justification into their classrooms and compares this set of purposes with those documented in the research mathematician community. Results indicate that the teachers viewed justification as a powerful practice to accomplish a range of valued classroom teaching and learning functions. Some of these purposes overlapped with the purposes in the mathematician community; others were unique to the classroom community. Perhaps surprisingly, absent was the role of justification in verifying mathematical results. An analysis of the relationship between the purposes documented in the mathematics classroom community and the research mathematician community highlights how these differences may reflect the distinct goals and professional activities of the two communities. Implications for mathematics education and teacher development are discussed.

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Justification is a practice at the heart of mathematics. As a disciplinary practice, justification has many purposes: it is used to validate claims, illuminate or provide insight into a result or phenomenon, and systematize knowledge, among others (Bell, 1976; de Villiers, 1990, 1999, 2002; Hanna, 1990, 2000). We know much less about the role of justification in K-12 classrooms (when it is present). Justification may be used in classrooms for purposes similar to those of mathematicians, but it may also play a role in other classroom-relevant aims or purposes (Knuth, 2002b; Staples & Truxaw, 2009).

Our focus on justification is not derived exclusively from its import as a disciplinary practice, but also from its role as a learning practice (Cohen & Ball, 2001). As a learning practice, justification is a means by which students enhance their understanding of mathematics and their proficiency at doing mathematics; it is a means to learn and do mathematics. There is empirical support for this connection as students in classrooms where they are prompted for their mathematical rationales express more complex and higher levels of mathematical thinking (Hiebert et al., 1997; Wood, Williams, & McNeal, 2006) and demonstrate greater student learning outcomes (Boaler, 1997; Kazemi & Stipek, 2001). Furthermore, classrooms that engage students in justification may support more equitable outcomes among heterogeneously grouped, diverse populations (Boaler, 2006; Boaler & Staples, 2008).

Currently, there is a general dearth of justification in US mathematics classrooms (Jacobs et al., 2006), even when teachers are implementing proof-related tasks (Bieda, 2010). This state of affairs is clearly a cause for concern. A deeper and more nuanced understanding of the practice of justification in K-12 mathematics classroom communities is critical if we are to expand the presence of this practice and meet the current visions of reform documents which increasingly emphasize the

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importance of justification (e.g., Common Core State Standards [CCSS], 2010; National Council of Teachers of Mathematics [NCTM], 2000). Specifically, we need to better understand how teachers think about justification in the context of their work. Teachers are deliberate actors, and their conceptions of the subject matter, aims and purposes, can play an important role in shaping their classroom practices (Fennema & Franke, 1992; Knuth, 2002a, 2002b).

In this paper, we explore the purposes of justification in middle grades classrooms. We first discuss justification and the potential relationship between the practice of justification in the mathematics classroom community and the research mathematician community. Next, we review literature on what is known about the purposes of justification in each community—the classroom community and research mathematician community. We then report findings from our study with a group of 12 middle school teachers, highlighting places of confluence and divergence between the purposes identified by the teachers and the purposes documented in the literature about the research mathematician community. Finally, we explore factors that contribute to the overlap and uniqueness of each set and discuss implications of these findings. For ease in communication, we subsequently use the term mathematician community to indicate the research mathematician community, and specify when we intend other communities of mathematicians.

In addition to documenting teachers’ perspectives, we compare the purposes of justification of these mathematics teachers with the purposes documented for research mathematicians for two reasons. First, a sizable set of research on justification in classrooms uses the purposes of the mathematician community as a frame of reference (e.g., de Villiers, 1999; Hanna, 2000; Knuth, 2002b). Consequently, this framing provides us the opportunity to contribute to this body of literature. Second, it is important to document when valued purposes or practices of a disciplinary community are and are not a vibrant aspect of the classroom communities in which that subject is taught. Such documentation is the first and necessary step toward making sense of why these similarities and differences may exist, evaluating if these differences are appropriate or desirable, and subsequently making adjustments to practice and/or our conception of alignment between communities. There is a need to make connections between these two communities of practice, and at the same time, to acknowledge that they serve different purposes which may shape their justification practices.

1. Justification and proof

Every discipline (and community) has its own standards for what counts as justification and, correspondingly, what is required to establish a conjecture or theory as a (working) truth. For example, in science, a strong empirical foundation is needed, as well as coherence with the prevailing scientific theories (which have been established based on empirical evidence) (Kuhn, 1962). In mathematics, establishing a new result generally requires a rigorous deductive argument, presented following agreed upon conventions, that demonstrates the truth of a mathematical claim, that is, a proof.

Theorists and researchers have no single, agreed upon definition of proof (CadwalladerOlsker, 2011; Jaffe, 1997), or of related terms such as informal proof and justification, although some argue that there is a fair amount of agreement on the definition of formal proof (Balacheff, 2002). For the purposes of this inquiry, we have chosen to use the term justification and define justification as an argument that demonstrates (or refutes) the truth of a claim that uses accepted statements and mathematical forms of reasoning. Our definition of justification is similar to Stylianides’ (2007) definition of arguments that function as proofs in a given classroom community. It differs in that we do not focus on the community aspect, nor do we try to make judgments about what forms of reasoning and representations are within the conceptual reach of the community engaged in the proving process.

Note that the type of reasoning used in the argument must be a mathematical form of reasoning (Larsen, McCaffrey, & Staples, in preparation; Larsen et al., 2011). Consequently, this definition excludes arguments that use reasons such as “because John told me” or “because that’s what you said was right yesterday.” These are not mathematical forms of reasoning. The definition, however, does permit arguments such as “That’s what we proved yesterday” which is an appeal to previously established results, and is a mathematical form of reasoning and used consistently in mathematics. It also permits empirical or example-based reasoning as a mathematical form of reasoning. Although in many instances, an empirical argument will not demonstrate the truth of a claim, this form of reasoning can be used to prove a claim depending on the question. For example, if students are asked to find the perimeter of the 10th and 50th figures in a pattern, an empirical argument, where the student counts, is valid.

Both the process of justifying and also the end point of having constructed a justification are relevant for thinking about the purposes and value of justification in the classroom. The same ideas play out when discussing proof and proving. As CadwalladerOlsker (2011) describes, “Proving is a process, which may include arguments and trains of thought which ultimately lead nowhere. The proof, which is the result of this process, will not include such dead ends” (p. 39). The value of proof, however, encompasses both. For example, a documented role of proof is discovery, but that discovery can come about through the process of proving— with its dead ends and false starts which evolves into a new discovery.

Our notion of justification is consistent with the Common Core’s (CCSS, 2010) description of the key practice Construct viable arguments and critique the reasoning of others. The document states the following: Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. (pp. 6–7)
Although our definition does not include the requirement that justifications also enlighten or promote understanding, we highly value this aspect of justification. This characteristic makes justification a learning practice, as it promotes understanding among those engaged in justification—both the individual offering a justification and the audience of that justification—in addition to pursuing a demonstration of the truth of a claim. This characteristic, as will be demonstrated, is also extremely important to teachers. Consequently, it is almost always the case that when we discuss justification in relation to teachers’ views, that we are considering justification as a practice that promotes understanding.

We use the term justification instead of proof as we find it more appropriate for discussions about middle grades mathematical work. The terms proof and justification may index different practices or concepts for teachers (Knuth, 2002b). The word proof has particular connotations and may conjure images of the two-column proofs common in courses on Euclidean geometry, or a high level of formality or rigor, which may not be seen as appropriate for one’s students (Knuth, 2002b). Justification captures the same goal of proof and proving (an examination of the truth of a statement) and allows access to ideas relevant to the practice without excluding ideas that may get ignored when the word proof is used. In particular, using justification lessens the likelihood of a narrow focus on the level of formality or specificity of the form of the argument.

It should be noted that we did not define justification for participating teachers. We were interested in their purposes for justification as they conceptualized it in the context of their classrooms. (See Thanheiser, Staples, Bartlo, Heim, and Sitomer (2010) for a discussion of characteristics of teachers’ definitions.)

2. Prior literature

The literature on both proof and justification is relevant for our understanding of the practice of justification in mathematics classrooms, and we examine research on both in this section. In reporting others’ work, we use the terminology used by the researcher(s) conducting the study (e.g., justification, proof). When discussing our study, we use the term justification, which we intend to encompass proof and proving, as noted above. Before reviewing the literature on the role of proof in the mathematician community and the mathematics classroom community, we consider the potential relationship between the practice of justification in these two communities.

2.1. What should be the relationship between justification in the K-12 mathematics classroom and in the discipline?

Previous research on justification in K-12 classrooms has generally assumed there should be a “match” between the nature and purposes of justification in the mathematician community and the mathematics classroom community. This stance is reasonable as the subject of study in mathematics classrooms is mathematics, and thus one would expect the practices and standards of a community studying mathematics to reflect those of the mathematician community. As Herbst and Balacheff (2009) note in relation to this common-sense stance, “since a notion of proof exists in the discipline of mathematics, it might be entitled to exist in classroom activity. And if it were to exist, it would be expected to exist in a form that is accountable to, if not compatible with, how it exists in the discipline” (p. 43). This stance is aligned with Bruner’s well-known view that any discipline can, and should, be represented authentically in the classroom (Bruner, 1960).

On the other hand, these two communities have different overall goals and thus one might expect important differences across these contexts. The finding that the practice of justification is generally not present in 8th-grade classrooms in the United States (Jacobs et al., 2006) lends support to the idea that the classroom community may not directly reflect the mathematician community—and begs the question of why this is the case. The mathematician community aims to expand the field of mathematics and produce new disciplinary knowledge. The mathematics classroom community aims to support novices’ learning of mathematical ideas and results that are new to those novices but may be known to the mathematician community. The work of teachers towards this end is guided by curriculum frameworks and other documents that explicate the mathematical knowledge and proficiencies students should master at each grade level. Such differences in overall goals may shape the purposes of justification in each community (Herbst & Balacheff, 2009; Herbst & Brach, 2006).

Hanna (2000) acknowledges that, at least to a degree, the overarching goals of the mathematician and mathematics classroom communities are different. She contends that the purposes of proof in the mathematics classroom community should reflect those of the mathematics community, but argues that the emphasis placed on each purpose might differ between the two communities and reflect the broader aims of each community.

It is useful, when attempting to set out the role of proof in the classroom in a systematic fashion, to consider the whole range of functions which proof performs in mathematical practice. Proof in the classroom would be expected to reflect all of them [the purposes of the mathematician community] in some way. But these functions are not all relevant to learning mathematics in the same degree, so of course they should not be given the same weight in instruction (de Villiers, 1990; Hersh, 1993). (Hanna, 2000, pp. 7–8, emphasis added)

Hanna further asserts that “in the classroom the key role of proof is the promotion of mathematical understanding” (p. 5), indicating the weighting she sees appropriate. Hanna’s approach maintains the primacy of the set of mathematician purposes and suggests a rebalancing of these purposes for classrooms.

As we seek to understand the role of justification in the K-12 classroom, we take the stance that the practice of justification should play a prominent role in mathematics classrooms, and one that reflects disciplinary practice. However, we would like to problematize the unexamined assumption, described above by Herbst and Balacheff (2009), that justification should play...
a central role simply because it is important to the discipline of mathematics. Assuming that the mathematics classroom community should mirror the mathematician community potentially positions justification as a practice valuable if one is on a path towards increased participation in a mathematical community, or perhaps for a liberal arts education; however, it does not offer a rationale for the inclusion of justification more broadly – as important for all students and for all teachers to incorporate across grade levels in their mathematics classrooms. Indeed, we know that teachers often only engage students considered “more able” in critical thinking and high order thinking practices such as justification (Raudenbush, Rowan, & Cheong, 1993; Zohar, Degani, & Vaaknin, 2001). We would also like to challenge the narrow approach of considering, and therefore valuing, only those purposes extant in the mathematician community. Such an approach does not allow for a broad examination of purposes important to teachers for their work in mathematics classrooms.

2.2. What do we know about the purposes of justification in both communities?

2.2.1. The research mathematician community

The purposes of proof in mathematics have been explored by de Villiers (1990, 1999, 2002), Bell (1976), Hanna (1990, 2000), Rav (1999), Weber (2008), Weber and Mejía-Ramos (2011) and others. (For a recent comprehensive review, see Yopp (2011).) We focus on a subset of purposes of proof in the mathematician community offered by de Villiers (1999). Although others are reported in the literature (e.g., aesthetic, intellectual challenge, learning new methods), this set (shown in Fig. 1) is well documented, appears consistently, and is recognized as highly influential (e.g., Mejía–Ramos & Inglis, 2009; Weber, 2010a; Yopp, 2011).

It should be noted that this set of purposes was developed in relation to the work of research mathematicians as professional mathematicians and not their work as instructors of undergraduate or graduate mathematics, which has been the focus of some more recent literature (e.g., Weber, 2010b; Yopp, 2011). de Villiers and Hanna both use the term proof in their work. As noted above, we use the term justification when considering the purposes of this mathematical work.

The role of verification is unsurprising. Those who document the purposes of proof in the mathematics community argue, however, that proof is not only about verifying or determining that a claim is true. It can serve many other important purposes in the community. Reflecting on this set of roles, we can see that each of these roles keeps the community vibrant and growing. Verification, systematization, communication and incorporation relate to the important activities of establishing, disseminating and archiving new knowledge – critical processes for any field. Explanation and discovery relate to expanding the capacity of the community to generate new results in the future. Providing insight and inventing are both means by which the community increases its knowledge and set of available resources to build and advance the field even further.

A note on verification is warranted. de Villiers describes this purpose as comprising two facets, that of verifying a result – meaning providing a mathematical argument for a claim – and that of conviction – convincing an individual, other person, or group that a claim is true (de Villiers, 1990). These two facets can be quite distinct, as one can be convinced of a result without having verified it, and one can verify a result, but still not be personally convinced of its truth (unless one “trusts” a mathematical system). In our use of verification, we focus on the former, primarily because of its importance in mathematics (eventually proofs must prove), and explicitly note when we are discussing verification in terms of personal conviction.

2.2.2. The mathematics classroom community

There seems to be two main findings concerning the classroom community and the role of justification in that community. The first finding is that, when present, verification is often the only (purported) purpose of proof to which students gain exposure. Even when proof is addressed and the role of verification highlighted, students may not come to appreciate this purpose, as proof is presented or perceived as a ritualistic process (Herbst & Balacheff, 2009; Schoenfeld, 1988). These critiques are generally aimed at undergraduate courses on proof and high school curricula which relegate proof to a topic of study in geometry. Students are often asked to prove results that seem obvious or trivial, and proof is presented as a mechanical exercise and does not spawn from, or lead to, meaningful activity. Schoenfeld’s (1988) classic study demonstrates

Verification (concerned with the truth of a statement)
Explanations (providing insight into why a statement is true)
Systematization (the organization of various results into a deductive system of axioms, major concepts and theorems)
Discovery (the discovery or invention of the new result)
Communication (the transmission of mathematical knowledge)
Incorporation (incorporating a well-known fact into a new framework)

Fig. 1. Purposes of proof [justification] in the mathematician community.
this well, as high achieving students – many of whom were very successful on the state’s end-of-course mathematics exam – had no sense of proof work as anything other than mechanical and algorithmic.

The second finding is the general absence of justification in K–12 math classrooms. In a random sample of 50 videotaped lessons of 8th-grade US mathematics classrooms, Jacobs et al. (2006) found no examples of lessons that involved proof. Furthermore, they found no instances of developing reasoning, making generalization, or using counterexamples, although two lessons had evidence of deductive reasoning. Even in a “best case scenario” where teachers used a curricular program that included proof-related tasks, and had been trained in the program, instances of student justification were relatively rare (Bieda, 2010). Bieda (2010) studied 7 middle school teachers using the Connected Mathematics Program (CMP) (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998/2002/2005). Across 49 lesson observations, only 28 student responses included an argument considered to be proof-related. Given that the observations were exclusively of lessons where proof-related tasks were implemented, these results reinforce claims about the dearth of proving activity in middle grades classrooms.

Some prior studies have aimed to unpack teachers’ views of justification and its role in their classrooms. Knuth (2002a, 2002b) studied 17 secondary mathematics teachers (2 middle school and 15 high school) and interviewed the teachers about the role of proof for their teaching. Knuth (2002b) found that teachers reported five roles for proof in their classrooms: Developing logical thinking skills, Displaying thinking, Communicating mathematics, Explaining why a statement is true, which focused on showing derivations to motivate formulas, and Creating mathematics knowledge. Knuth was struck by the lack of reference to the explanatory power of proofs, that is, the role of proof in “promoting insight of the underlying mathematical relationships” (p. 80), which is highlighted in the literature as potentially the most valuable role of proof in K–12 classrooms, as discussed above.

Staples and Truxaw (2009) explored 24 grades 4–9 teachers’ notions of justification by asking teachers to identify which justification from a set of student work samples was the “best” justification and why. Teachers’ responses highlighted valued characteristics such as: the students’ work was detailed, a (previously taught) method was identified by the student (or readily identifiable), and the student was able to see or use the key relationship. The researchers concluded that teachers likely prefered justifications with these characteristics because such justifications gave them insight into what students had learned as a result of their instruction. Notably the teachers did not comment on whether the argument demonstrated the result was true (verification) as a valued criterion. Together, these studies suggest that there are purposes that teachers potentially attend to when asking students to justify that are not reflected in de Villiers’ set.

The research reported here inquires into teachers’ purposes of teaching with justification and the relationship between the purposes of these two communities – the mathematics classroom community and the mathematician community. We present data from the first year of a 2-year study of 12 middle school mathematics teachers who were actively working to incorporate justification (proof) into their classrooms. We focus on the purposes teachers identified for justification in their classrooms and why these purposes were important to them and reasonable in the context of their work. As a general approach, inquiries that look solely for classroom purposes that reflect the purposes of justification in the mathematician community background teachers’ work, what they are doing with respect to proof in their classrooms, and why. Foregrounding teachers’ work and their reasoning provides us insight into what teachers who are making deliberate efforts to incorporate justification are trying to accomplish when they engage students in this practice, and why it is, or is not, represented in particular ways in K–12 mathematics classrooms.

3. Conceptual framework

In approaching this research, we drew upon sociocultural frameworks and specifically the notion of communities of practice (Lave & Wenger, 1991; Wenger, 1998). A community of practice is a group of people who engage in a shared domain of human endeavor (joint enterprise) for a sustained period of time (Wenger, 1998). As noted above, the mathematics community and mathematics classroom community, though related, are distinct communities of practice and pursue distinct joint enterprises. This community-focused framework brings attention to these distinctions, prompting a careful examination of the nature of activity within each community in relation to its overarching goals.

The focus on practice provides a useful conceptual tool. A community-of-practice lens highlights the fact that any practice is locally constituted and constantly renegotiated (sustained or modified) in interactions among the community’s participants. A practice is customary within a group; it has tradition and history, and it accomplishes particular purposes. Thus, a practice such as justification, has no inherent form or meaning, but rather is given meaning within the context of the community. Different communities can sustain different versions of a practice. Justification, then, may have different purposes and forms across different communities.

de Villiers (1990), in his description of the role of proof, highlights the role of community in relation to its practices of proving, thus supporting this view. He positions the establishing of a proof as ultimately a social process. Citing Davis (1976), de Villiers argues that what counts as a proof is not an absolute across communities, or even within a community. “Proof as a form of social interaction . . . [and] also involves the subjective negotiation of not only the meanings of concepts concerned, but implicitly also of the criteria for an acceptable argument” (p. 22). What justification or proof is, and the purpose(s) it serves in a community then, are not imposed from elsewhere, but rather co-constructed within the community of practice (Cobb, McClain, & Gravemeijer, 2005; Yackel & Cobb, 1996). Consequently, in approaching this work, we centralized the notion of community and sought to understand, from the perspective of those members of the community, the role and value they saw for the practice of justification in their mathematics community.
As noted above, important to our perspective is that justification can be seen as both a disciplinary practice and a learning practice. As a disciplinary practice, it is a process by which mathematicians create and validate new knowledge (Davis & Hersch, 1981; Devlin, 1994; Lakatos, 1976). As a learning practice (Cohen & Ball, 2001), it is a means by which students and others augment their understanding of mathematics and their proficiency at doing mathematics.

Drawing on a community of practice perspective, we argue that the two communities – the mathematics and the mathematics classroom – are fundamentally different – with different purposes, goals, roles, accountability structures – and that these differences play a critical role in shaping the work teachers do with respect to justification and the value they identify for incorporating it into their classroom. In turn, this shapes the nature of justification in the mathematics classroom community and what students might learn about this important practice.

4. Data methods and analysis

4.1. Participants

Participants in the study were 12 middle school mathematics teachers (grades 7 and 8) taking part in JAGUAR, a project funded by the National Science Foundation. JAGUAR stands for Justification and Argumentation: Growing Understanding of Algebraic Reasoning. Teachers recruited for the project had full certification (four at the secondary level), had taught middle school mathematics for a minimum of 1 year, and had participated in prior professional development experiences orienting them to incorporate student discourse. We aimed to recruit teachers who had developed a level of sophistication in their pedagogy that produced a classroom culture classified as ‘strategy-reporting’ or ‘inquiry/argument’ (Wood & Turner-Vorbeck, 2001). In such classroom cultures, students’ thinking is regularly elicited and publicized, and is positioned as valuable to the lesson. All but one of the other teachers had extensive exposure to related ideas (e.g., teaching for higher-order thinking) through professional development or prior participation in research projects, or both. Half of the teachers had previously participated in intense professional development geared towards promoting student discourse in mathematics classrooms. In general, this group of teachers already expected their students to participate in classroom discourse. The 12 teachers taught in five districts in two states and had 2–29 years of experience.

In committing to the JAGUAR project, the teachers agreed to work actively on ideas related to justification in their practice and to collaborate with project personnel to unpack the nature of justification in middle grades mathematics classrooms. We do not expect that these teachers are representative of the larger population of middle school mathematics teachers. Rather, this was a purposive sample (Yin, 1994) to enhance our ability to examine the role of justification in middle grades classrooms.

4.2. Data and analysis

A range of data informed the analyses reported here. Initial themes and categories related to the purposes of justification in the mathematics classroom were developed by reviewing documents and video taped discussions from a week-long summer course and two Saturday working sessions, transcripts of teacher interviews, videos of lessons, and teacher reflective journals. A list of all purposes was generated and then thematically organized using standard qualitative methods of open coding and constant comparative method (Strauss & Corbin, 1998).

We then conducted a systematic analysis using these themes on two data sources for confirmation and refinement. We revisited four discussions (~1 h each) from the summer course and work sessions regarding the role of justification in their classroom and reviewed teacher reflective journals (36 total) on the implementation of three tasks that were deliberately designed to engage students in justification. These tasks (e.g., the Hexagon task) included prompts that asked students to explain why or justify mathematical claims or results.

In the working session discussions, teachers were asked to identify and share the purposes of justification in their classrooms. These discussions lasted approximately 1 h each and revealed a wide range of potential purposes of justification. For the reflective journals, teachers wrote about justification prior to the lesson, responding to questions such as “What role will justification play in this lesson?” and then reviewed the videotapes of their lessons and responded to an addition set of questions including “What, if anything, do you think students learned about justification as a result of this lesson?” (asked after the lesson). See Appendix A for the set of questions reviewed from the reflective journals. We reviewed three journals for each teacher on three justification tasks (common across all teachers).

In reviewing the reflective journals, we also looked for three additional purposes of interest by identifying text that indicated that teachers were interested in accomplishing the following goals: (a) developing students’ understanding of what justification is; (b) using justification to validate (prove/establish) valued mathematical results; and (c) teaching proof techniques or skills. These three themes did not appear prominent in our initial review of journals or discussions, but appear in the literature and are important to explore for understanding how teachers are using and positioning the practice of justification in their classrooms. Thus, we felt it was important to code for the presence of these purposes.

The summer and working sessions and teacher reflective journals offer two different windows into teachers’ practice. For the collaborative sessions, teachers could build on one another’s thinking and were pushed to clarify their meaning as others sought to understand the ideas in the moment. In contrast, teachers’ reflective journals are closer to practice than discussions in collaborative sessions, by which we mean they are produced as teachers actively reflected on specific instances
Promoting Conceptual Understanding (supporting student learning of mathematics content)

Fostering Valued Math Skills and Dispositions (supporting student learning of math-focused skills and dispositions e.g., using or connecting across multiple representations and perseverance in the face of a mathematical challenge)

Assessing – Displaying and Monitoring (providing the teacher with information to know what students have learned and adapt to the students’ needs)

Fostering Valued Life-Long Skills and Dispositions (supporting student learning of skills and dispositions for adult life or the workplace, e.g., developing confidence, the ability to take a stand and support it with evidence)

Managing Diversity (offering access to, or reaching, a wider range of students)

Influencing Social Relationships (shaping student-student interactions, and moving teacher away from the central authority role)

Fig. 2. Purposes of justification in the middle school mathematics community.

of practice instead of reflecting more abstractly on their practice in general. It should be noted that reflective journals are more variable, as teachers reflect on aspects of the lesson of their choosing, although structured by our prompts. We expect that different information may be elicited through other means, such as a structured interview about a taught lesson.

The first author and a research assistant independently reviewed the 36 journals (teacher by teacher), identifying which of the themes were present in the teacher’s reflective journal, and then discussed each teacher’s journals, recording a final summary. Six final themes were identified as a product of this process (see Fig. 2). All six themes were present in the working session discussions. Four of the six themes were more prominent in the journals. We did not discard the two less prominent themes however as our goal is to offer an expansive picture of the potential purposes of justification in a middle school mathematics classroom.

One of the work session discussions served as a member check (Lincoln & Guba, 1985) on the themes to enhance validity of the results. A group of four project teachers were presented with a list of the set of purposes of justification in the classroom, as well as a list of purposes of justification (proof) in the mathematician community. They were asked to identify where they saw the two lists overlapping, as well as any purposes they felt were distinct to one community or the other. This process led to a refined understanding of teachers’ views of the purposes.

In reporting the results, we share major themes and then offer supporting evidence from the broader set of data, as appropriate. Teachers are reference by identifiers T1, T2, T3, etc. The quotes provided are from the working session discussions. We acknowledge that a different set of purposes may be identified by a different group of teachers. We also acknowledge that the prominence of these purposes for individual teachers may vary.

5. Results

The results of this study indicate that middle school teachers, who are actively working to incorporate justification into their practice, use justification for a wide range of purposes, some of which overlap with the mathematician community and some of which are distinct. We report six purposes, reflect on the correspondence between these purposes and the mathematician community’s purposes, and analyze the teachers’ role and context in an effort to make sense of the set of purposes identified. We do not intend this set to be comprehensive or unique, as our group of teachers is not typical and work with other teachers or further analyses may identify additional themes and other useful categorizations. Rather, we document the existence of purposes identified by this group of teachers in order to paint a picture of the potential purposes of justification in middle grades classrooms. Six purposes of justification in the middle school mathematics community are listed in Fig. 2.

We explicate this list by discussing each purpose, offering supporting examples from the data, and then discussing the purpose in relation to the literature.
5.1. Promoting conceptual understanding

One important purpose of justification in the classroom was to promote or deepen students’ learning, particularly of concepts. Teachers noted the process of justification led to deeper understanding because it required students to wrestle with the main ideas, make connections, and gain new insights. The means by which this happened were many, ranging from hearing other students explain to students clarifying their own thinking as they tried to articulate their ideas. Centralized in this process was the individual students’ construction of knowledge whereby justification offered students the opportunity to figure out for themselves how and why something worked. The following quote offers one example of the teachers’ views: “Justification pushes students beyond a procedure to a deeper understanding of the math. In order to justify their thinking, they have to justify not just the hows, but get to the whys of what they’re doing” (T1). The theme aligns strongly with teachers’ explicit charge: to teach middle school students mathematics content.

The purpose of promoting or deepening students’ learning is anticipated in the literature. We see this as akin to Hanna’s category Explanation elaborated as promotes insight into a phenomenon. Although she positions the mathematician community as the collective recipient of this insight, it can be understood on the individual level as well. She also anticipates that this role is prominent in K-12 classrooms, as it supports student learning.

This purpose is also related to the purpose of Incorporation (incorporation of a well-known fact into a new framework). Teachers noted that justification allowed students to build on prior knowledge, connect two ideas in a new way, and review other course material (which promoted learning more generally, not just conceptual understanding). It helped “keep math connected” for students.

5.2. Fostering valued mathematics skills and dispositions

The teachers also identified the growth and development of other math-related skills and dispositions as an important purpose of justification. For example, teachers saw that engaging students in justification helped students develop their communication skills and representational skills (such as generating and using graphs, tables and symbols), as well as making connections across representations. Across four work session discussions, all five process standards (NCTM, 2000) were noted. For example, representation was indicated with one teacher’s comments that “because justification usually involves some sort of representation, like symbolic and graphical” it gave her the opportunity to engage students in “developing and practicing” different representations (T8). We note that there are other means to work on representational fluency, so this is not a unique contribution of justification. Justification, however, was seen as a useful tool for helping reach the valued end goal of representational fluency.

Focusing on having students justify as a means to develop student communication skills, a teacher offered the following.

T2: I think it helps them develop their ability to communicate their thinking. They can hear it from others, yeah sure, but putting it in their own words.

Related to this aspect, another teacher (T6) commented on the “language focus” of justification, as justification created an authentic need to communicate and use mathematical terms in context.

We do not see an analogue in the mathematician community to this set of outcomes, perhaps because there is little need in the mathematician community to focus on the development of such process skills. These are tools of the field and it is assumed that mathematicians are proficient with these practices otherwise they would not be members of that community. Other skill development, however, may result from engaging proof, as researchers have documented that mathematicians read proofs to learn new techniques and forms of representation (Rav, 1999; Weber & Mejia-Ramos, 2011).

5.3. Assessing – displaying and monitoring

Justification was seen to play a critical role in creating a venue for students to display their understanding so that teachers could monitor the degree to which students had moved towards desired learning goals. This theme encompassed formative assessment, including self-assessment, and summative assessment.

Teachers asked students to justify as a window into students’ reasoning about a particular idea. From this they gleaned information, useful for diagnostic purposes, about how the student, or class, was thinking about a problem. It helped teachers “pinpoint” where students were stuck, or where their misconceptions might lie. As one teacher noted: “It helps to guide my next instructional moves. So based on a student, or the collective class’s justification, . . . it shows me where I need to go next” (T2). Teachers also found value in justification as self-assessment, as justification prompted students to reflect on their own work and identify their own mistakes. This overlaps with the first category reported, promoting conceptual understanding.

The emphasis teachers placed on justification as having students “show what they know” is particularly interesting to examine. Consider this teacher’s comment in the context of discussing justification.

T12: Interestingly enough, our district has gone to proficiencies, . . . And it’s now a kind of thing too, that proficient, or highly proficient, just showing an answer doesn’t give me any clue as to your proficiency or understanding. So that’s again, putting that responsibility back on the student to really show what they know.
This exercise of "proficiencies" was an exercise of displaying; it involved the student taking responsibility for displaying understanding through a response.

An interesting dimension of this assessing purpose was revealed in interviews when teachers were asked to determine whether a student response was or was not a justification. In answering this, rather than engaging the question, "Has the student demonstrated the result is true?" teachers seemed to engage the question, "Has the student demonstrated a full understanding of the key ideas we have been working on?" (i.e., did the student learn?) For example, one teacher claimed that a particular justification offered would be one for an 8th grader, as she would expect the 8th grade would understand the reasons behind a particular claim used in the justification, "but for a 6th grader, I want to make sure he knows why it's working." In these instances, it seemed teachers looked for different information than what might be required for a complete justification of the result (Thanheiser et al., 2010). This distinction is captured by considering what the teachers were looking for evidence of: that the students had learned the particular content, rather than the validity of the result. Staples and Truxaw’s (2009) study found similar results, as did Knuth (2002b) with his category displays thinking.

Reflecting on this purpose, one sees the reasonableness of this role for justification from the teachers’ perspective. Teachers are responsible for both developing and certifying students’ mathematical proficiency. By asking students to justify (produce an argument to verify), teachers gain the opportunity to “see” students’ thinking and gain important information regarding whether or not, and often to what degree, a student has demonstrated understanding of particular concepts that the teacher is responsible for teaching.

There is no clear parallel of this assessing purpose among the purposes of the mathematician community. Verification as a purpose in the mathematician community has an evaluative bent to it, as does assessing as a purpose, but they are fundamentally quite different. The focus of the mathematicians’ thinking is whether or not a result is valid; the focus of the teachers’ thinking is whether the student learned the material.

One point to raise is that the particular qualities or features of justification desired when the practice is being used for assessment may differ from when the practice is used for other purposes. Teachers demand a high level of explicitness in assessment situations and may not allow students to reference already established facts as the teacher wants to know whether these facts are memorized facts or understood. This issue is important because it may significantly influence what students think a justification is or accomplishes. The emphasis on displaying knowledge may require students to include non-essential details or information and not build on prior results, as they to re-prove the results to demonstrate their understanding of the underlying mathematics. Similarly, students might use particular approaches such as multiple representations, even when not necessary for the purpose of justifying, to demonstrate connections or facility working across representations. Preliminary findings from some of our other data indicate this may be the case. Again, this follows from the idea that the meaning of a practice is locally negotiated and constituted.

5.4. Fostering valued life-long skills and dispositions

Beyond mathematics, justification was seen to play a role in promoting the development of valued life skills and dispositions – goals important for students not as people who do or use mathematics, but as life-long learners and/or future adults. The teachers noted that engaging students in justification fostered student perseverance, independence, critical thinking skills, general communication skills (written and aural) and the habit of mind to support one’s ideas or request that of another.

T4: It’s like real world skills that they need for problem solving or to kind of learn. It’s like learning skills that happen to come out nicely through mathematical justifications. Like when are you going to make mistakes? Well, we’re going to make them when you’re trying to justify something... So here’s the opportunity to learn that we learn from mistakes.

T12: Using it [justification] to develop critical logical thinking, something that’s going to be life long skill. It doesn’t have to be restricted to math. It’s really something you’re trying to create...[to] have the willingness to have a dialogue with each other, and not just a critical, “what did you do?” But really start to have an interaction of, “I don’t know where you got that from”, or “can you explain that to me?”

These skills and dispositions of course can also be seen as critical mathematical skills – they support students’ mathematical work and are characteristic of students who are mathematically proficient. The teachers’ emphasis, however, was on skills for life-long learning and/or success.

Communication was also noted as a life-long skill. This included, but extended beyond, communicating mathematically. It was seen as a more general process or outcome, applicable to and important for one’s adult life. Referencing her own professional work, one teacher made the following comment:

T9: Because when you’re done with a piece of work, ... whatever I do, I want it to stand by itself without me having to stand there and explain it. ... And I want kids to be able to do that too – to be able to put something down well enough, whether it’s on paper or whatever, that it can stand by itself without them having to sit there and explain it.

In discussing broad goals, teachers also brought in connections with other subjects, noting how each subject contributed to a student’s ability to communicate, make an argument, or become a critical thinker – goals that transcended math class.
This purpose of fostering life-long learning skills and dispositions also seems unique to the mathematics classroom community. There is an element of overlap of this with the purpose of communication, although the spirit of communication expressed by these teachers (a life skill) and purpose intended by those writing about the mathematician community seem quite different. For the mathematician community, the communication purpose seems to be primarily a means of disseminating and archiving knowledge. There are particular conventions for communicating and means of dissemination within a community (e.g., journals). In part, being a member of the mathematician community means that one is knowledgeable about these conventions and knows where and how to access the archived knowledge of the community. By contrast, these teachers were interested in developing each individual student’s capacity to communicate his or her ideas, both within the mathematics class and beyond. For some teachers, communication overlapped with other life skills goals such as fostering independence.

Although teachers are not explicitly charged with developing these life-long learning skills and adult skills, these teachers not surprisingly thought holistically about their work as educators and aimed to do more than teach the prescribed mathematics content to their students. They pursued goals that they felt could be fostered in the mathematics classroom, supported by the practice of justification, but their purposes transcended mathematics learning.

5.5. Managing diversity

Justification provided teachers other affordances for their instruction as well. They saw it as a means to make them more effective with a range of students. Justification inherently differentiated learning, as when asked to justify, students could approach a task in any number of ways and take it as far as they could, offering justifications at different levels of sophistication. It made it so not everyone had to move “lock step,” but rather all could be pushed and learn even at different levels, as this quote from a teacher reveals. This teacher is referring to implementing a justification task.

T11: This will push some kids, in a heterogeneous classroom, beyond into a generalization, and this problem will push these kids here still to a justification [but not a generalization], but I can push the others beyond where the others are.

Following on this idea, other teachers added that justification tasks— at least those they designed— often had “lots of points of entry” (T12), which also was beneficial in a heterogeneous group. Regardless of the level the students might be at “there’s still justification happening at each level” (T11).

Teachers also saw incorporating justification as a way to “reach every student.” Having students share many approaches to one problem offered students multiple paths to understanding. They were bound to understand at least one approach, and teachers noted that sometimes students had better ways of helping classmates make sense of ideas than they did.

T9: Maybe kids don’t see it one way, but someone else has a different approach, [that] can actually really help them.

For approaches that many students did not understand, the teachers felt the exposure to these approaches was still productive for student learning, laying some foundation for future work. Note that this aspect of managing diversity also overlaps with the first category promoting conceptual understanding, but is reported here for its role as a pedagogical affordance to the teacher in organizing instruction with attention to a diversity of learners.

It was unclear from these data whether justification played a unique role in creating opportunities for students to share multiple approaches, or whether any opportunity— through justification or otherwise— to share multiple approaches was valuable. For these teachers, justification and multiple approaches seemed tightly linked, and for some, perhaps one and the same. Analysis of additional data is needed to further clarify this relationship. Regardless, teaching with justification made ample space for teachers to elicit multiple approaches and representations, and thus leverage that aspect of student work to help manage the diversity in their classroom.

The theme of managing diversity also showed up in relation to the assessing (monitoring) function. With a diverse group, it was very important to the teachers to elicit information on students’ understanding to inform their next steps and adjust instruction.

5.6. Influencing social relationships

Teachers noted that justification could influence the social relationships or impact the social dynamics in their classroom. It was felt that, if mathematics becomes the authority, it can challenge the social status quo and can shift the classroom from a social system organized around the popular to those who are good thinkers. One teacher stated the following.

T7: When you add the word middle school, now I’m thinking a little more socially. I feel like justification can have a couple social functions. And one of them is sort of valuing, like, how we decide who’s right. Instead of it being, you know, the teacher’s right, or this kid is popular, so they’re right, it has a social function of, you can prove it, so therefore, I don’t care if you’re the least popular kid in the class. You know, you have to be able to back up your ideas. Everybody has to justify what they’re saying. … I think it takes some of those power relationships out a little bit.

Similarly, teachers talked about justification “empowering” students, which potentially shifted the dynamics so that the teacher was not the only one who knew. Students might learn to trust that they can figure things out for themselves, or that they could use their peers as resources.
This theme was not as prevalent in the reflective journals, perhaps because of the nature of the questions asked of teachers for their reflection. It was however present in discussions and is worthy of note given the different perspective it offers on the value of justification for supporting a range of desirable outcomes. Other instruments may elicit this theme more or less strongly.

This purpose also has no parallel among those reported for the mathematician community. Although it is possible that this outcome is operative in the mathematician community (where potentially lower-status newcomers can intellectually go head-to-head with established mathematicians), it is not articulated as a purpose. By contrast, this aspect has received extensive attention in the mathematics education literature in discussions about student agency and positioning students as knowers and doers (not receivers) of mathematics (e.g., Boaler, 2000; Gresalfi, 2009; Gresalfi, Martin, Hand, & Greeno, 2008).

5.7. Purposes of the mathematician community not identified

5.7.1. Verification

Notably, teachers generally did not identify verification as a purpose of justification in their classroom communities. That is, establishing mathematical results or ideas as true was not regarded as a goal of justification. Given the centrality of this purpose to the mathematician community, we discuss some of the teachers’ views in more depth.

Ideas potentially related to verification were mentioned, for example, teachers noted that students can check their own work, or do not have to rely on the teacher. Such examples hint towards notions of personal conviction that de Villiers (1990, 1999, 2002) discusses when he talks about the purpose of verification. In these contexts, however, the valued goal was fostering student independence and self-reliance (life-long learning goals), or shaping social relationships (as described above), and not establishing a result as true for the community to then use and build upon. We attribute this discrepancy to the different overall aims or goals of these two different communities – the mathematician community and the mathematics classroom community. Importantly, the classroom-based purposes teachers did identify harness the fact that justifications aim to verify, which may be the driving force behind why justification is such a powerful pedagogical tool. That said, the verification aspect of justification was backgrounded for the teachers.

During the member check, the teachers, while committed to having students justify and moving students towards greater proficiency with justification, articulated reasons why the purpose of verification might be less prominent in a mathematics classroom. The teachers noted that in the classroom students had other resources, including the teacher, to turn to for verification. de Villiers (1999) echoes this idea arguing that there is no driving need for verification in the classroom community. By contrast, in the mathematician community, members are establishing new claims, so verification must play a central role. As one teacher noted:

T4: Like verification, concerned with the truth of a statement, is really what mathematicians are concerned with because, where they are, they’re forging new ground. So no one is telling them that, “yes, this is true.” So they have to be very concerned with verification because if we take it to be true and it isn’t true, then we’re going to develop new ideas off of it, and those won’t be true because this one isn’t true.

This quote reveals this teacher’s perceptions of some important differences of the constraints and affordances (Greeno & MMAP, 1997) between the mathematics classroom community and mathematician community, namely potential “locations” of authority. This difference is relevant for understanding justification as an act of verification, and why this purpose would play a prominent role in the mathematician community and might not the middle school mathematics classroom community.

We recognize that situations may arise where a student proposal is made and neither the teacher nor students know whether the proposal is true. This situation creates an authentic context for justification in the mathematics classroom if teachers create space to explore and examine the truth of the claim. Similarly, when students offer incorrect answers, the purpose of verification may be prioritized. More research is needed to explore how teachers manage these situations and the degree to which such instances spawn justification activity focused on verification that teachers frame as being undertaken for the purpose of verification.

Teachers also noted that they engaged students in justification activities at various points in the learning process, specifically, as students were still developing their own mathematical ideas. Thus teachers involved students in justifying before the students were equipped with the requisite understandings to verify a larger claim. Consequently, there were times when the students were not (yet) in a position to fully rely on themselves, or peers, to verify their method and/or the result. As one teacher noted, as students learn, “they need the teacher to validate because they don’t ‘trust’ the method yet” (T10). Students however, through engaging in justification, can develop their mathematical understandings, which then better positions them to use justification for verification. Indeed, it may be that the main goal of a lesson is to deepen students’ understanding of ideas that come into play when students are asked to produce a justification in response to a given task or prompt, and less to verify the claim itself. Thus, a teacher may incorporate justification into a lesson to reach other valued learning goals rather than for the purpose of verifying a claim. Yopp (2011) documents a similar phenomenon, highlighting how justification can build “horizontal” connections across ideas. It should be noted that, regardless of the main pedagogical goal, students will necessarily be simultaneously learning about and negotiating what constitutes an acceptable mathematical justification.
On a related note, the teachers acknowledged that at times they asked students to move forward without requisite foundation, or to take something as true without demonstration, perhaps for pedagogical or practical reasons. Students then potentially were asked to make sense of new ideas when they still had gaps in their prior understanding, or when some foundational ideas were still developing. Consequently, students were not in a position to produce a compelling justification of a result because the base information was taken as given without justification (or understanding). In these cases, justification may still be used to reach desired pedagogical goals, although it would not serve the purpose of verifying claims.

5.7.2. Discovery

Discovery was not a purpose of justification for this group of teachers. The idea of discovery seemed to emerge only as a step in the learning process, and not an outcome of engagement in the process of justification. Teachers often wanted students to conjecture that some relationship held (i.e., they designed the task to allow students to discover certain patterns and make conjectures), but did not implicate the process of justification itself as leading to discoveries. Rather it was more experimentation and pattern searching that led to discovery. The teachers knew in advance what mathematical relationships they hoped would be discovered, and the process was often done inductively. This classroom process contrasts with the description of discovery provided by de Villiers (1990) where engaging in the activity of proving led to new discoveries (e.g., by assuming the parallel postulate did not hold, in an effort to engage in proof by contradiction, new geometries were opened up).

5.7.3. Systematization

Systematization of mathematics was not a purpose of justification for these teachers. They were interested in having students develop a conceptually connected view of mathematics. Systematization, however, with its focus on mathematics as a system of knowledge, where new results are derived from prior results, using clearly established and referenced axioms, definitions, etc., was not indicated as a purpose of incorporating justification.

The purpose of systematization may not be in the foreground as teachers make decisions about engaging students in justification and what they want made explicit in a student’s argument. As noted above, in relation to the purpose of assessing, teachers may ask for students to re-establish an idea already vetted by the class. Similarly, as noted when discussing verification, teachers may ask students to take something as true with partial or without demonstration, as they attend to specific learning goals of their daily lessons. For example, a lesson’s learning goal may be: students will be able to “apply the Pythagorean theorem to find the distance between two points in a coordinate system” (CCSS, 2010, p. 56). Teachers may ask students to use the Pythagorean theorem in this new context regardless of the students’ understanding or awareness of how the Pythagorean theorem was derived from prior results and how it fits in with other results. Pacing guides or the particular ordering of content across a year may contribute to the challenges of teaching in a manner that allows for systematic building.

Systematization as a purpose also likely requires an appreciation and understanding of axiomatic systems, including the role of definitions, axioms, and theorems. Given the research literature indicating that few high school or college students reach a level of sophistication in their mathematical thinking to appreciate such systems (Burger & Shaughnessy, 1986; Harel & Sowder, 1998), this purpose seems an unlikely one to pursue or to achieve at a middle school level. That said, there could be elements related to systematization, such as understanding the role of a definition, that could reasonably be pursued.

6. Discussion

We have explored six purposes present in these teachers’ discussions and reflections, and identified where these purposes overlap with the set of purposes documented for the mathematician community. Specifically, to summarize, this group of teachers collectively identified the following purposes: promoting conceptual understanding, fostering valued mathematics skills and dispositions, assessing – displaying and monitoring, fostering valued life-long skills and dispositions, managing diversity, and influencing social relationships. We noted strong overlap with the mathematician community purposes of explanation and incorporation. Not present in the teachers’ set was the purpose of verification, systematization and discovery.

In this section, we reflect on justification as a learning practice to explain our overall finding of divergent purposes of justification between the mathematician community and the mathematics classroom community and to connect our results with prior literature.

6.1. Related, but distinct, communities of practice

Why are the purposes of justification in the mathematics classroom community reported by this group of teachers (and others) quite different from the set of purposes of the mathematician community documented in the literature? In reflecting on the two sets of purposes, it becomes clear that the purposes of justification is related to the overall purpose (enterprise) of the community. Mathematicians, who are engaged in disciplinary knowledge development, use justification (proof) in a manner that supports this work. Each of the documented purposes – verification, explanation, systematization, etc. – supports the community’s joint enterprise of archiving and expanding mathematical knowledge. Similarly, middle school teachers of mathematics, who are engaged in an educational enterprise that foregrounds developing students’ mathematical proficiency, use justification in a manner that supports that endeavor, and more. Each of their purposes – promoting conceptual
understanding, assessing, influencing social relationships, etc. – relates to the overall joint enterprise of their community – to develop their students’ mathematics or to support their students’ development into productive, independent adults.

These findings document the teachers’ commitment to purposes related to justification as a learning practice more than a disciplinary practice. The value of justification teachers identify is articulated not in terms of its import as a practice that allows a community to expand disciplinary knowledge, but rather for its import and effectiveness as a practice that is useful in helping them meet their obligations as educators, both mandated (e.g., content standards) and non-mandated (students will develop into productive, independent adults). The practice of justification – what it is and what it does – is shaped locally, on the ground, within the institutional (and cultural) context of schooling, in relation to the larger purposes of the community’s work.

6.2. Relationship to purposes documented in other literature

The findings reported here are not dissimilar to Knuth’s (2002b) analysis of 17 secondary teachers’ views of the role of proof in school mathematics. The roles he found the teachers committed to were indeed roles with consequence for their work as mathematics educators. Specifically, he reported that teachers valued proof for the way it displayed students’ thinking and for its role in developing logical reasoning skills. These learning outcomes link to the categories assessing – display and monitoring and fostering valued life-long skills and dispositions, respectively. Our findings are also in line with the results of Staples and Truxaw (2009), specifically, that teachers valued aspects of student justifications that offered teachers information important for their role as assessor.

Unlike Knuth’s teachers, however, this group of 12 middle school teachers did report that justification was important for promoting conceptual understanding or providing insight, which is linked to the explanatory nature of proof (de Villiers’ explanation function, also discussed extensively by Hanna (1990, 2000) and Hanna and Barbeau (2008)). This difference may result from Knuth’s explicit use of the term proof (and not the broader idea of justification) and/or the use of written proofs during his interviews, which may focus teachers’ attention on more formal proofs, or on proofs as products and not proving as a process or practice. This terminology and representation of proof work may index a static notion of a justification for teachers or particular professional activities, and may not prompt teachers to reflect on the range of teaching activities where the role of justification in developing students’ reasoning might be more salient. Future research might deliberately attend to the differences resulting from foregrounding particular language, as well as particular teaching activities and contexts (e.g., assessing vs. introducing new material), to further explore the role of justification and how the role of justification may vary by professional activity.

These findings, in conjunction with a recent study by Yopp (2011) of mathematicians’ views of the purposes of proofs in their own instruction, yield additional insights. In interviews with 14 research mathematicians (pure, applied, and statisticians), Yopp found that fewer than half of the mathematicians noted verification as a purpose of proof, further supporting findings that verification may not come to the fore as a core instructional purpose when teachers engage students, whether K-12 or undergraduate, in justification.

Yopp (2011) also documented a purpose he described as “using or including proof or proving to make meaning of mathematics other than the statement or theorem being proved” (p. 120). This description aptly captures the nature of activities the middle school teachers centralized in their work with justification. Specifically, teachers were less concerned with having students validate the particular result in question (e.g., that a number trick would work for all integers), and were more concerned with the development of particular connections and ideas that students engaged in the process of justifying (e.g., developing a deeper understanding of the distributive property by engaging the particular number trick). Yopp describes this as building connections in a manner that is “horizontal” as opposed to building in a manner that is “vertical” which taps into the axiomatic structure of mathematics, and which is what one does when proving a result. Future studies might explore this broader set of emerging purposes in the context of instruction more explicitly.

7. Implications

In this section, we reflect on how these findings suggest a strengthened rationale for including justification in middle grades classrooms – one that does not rely exclusively on its presence in the mathematician community. We also discuss the organization of professional development activities, and how those might more effectively support teacher learning and teacher change with respect to incorporating justification in the mathematics classroom.

7.1. A rationale for including justification in middle grades classrooms

We began this paper critiquing the argument that justification should be included in K-12 mathematics classrooms simply because it is a valued mathematics practice. The argument that it belongs in classrooms because of its value for a mathematics community apparently holds little weight with teachers. As noted, few middle school classrooms support the practice of justification. In our current educational context then, justification is, in practice, optional. A teacher may choose to include it, but it is quite possible to teach middle school mathematics without engaging students in this practice.

A more compelling and appropriate rationale attends to justification not solely as a disciplinary practice, but also as a learning practice. Justification is a powerful learning practice and pedagogical tool, and this may be readily recognized by
teachers. This group of teachers, as well as teachers in other studies, value justification for its role in promoting student conceptual understanding, assessing and monitoring learning, and helping them manage diversity. Other literature tells us that justification can promote more equitable outcomes (Boaler & Staples, 2008). Consequently, when articulating a rationale for including justification in middle grades classrooms – and across the K-12 curriculum – it is important to centralize the value of justification as a teaching tool and learning practice, and not just as a disciplinary practice. That is, it is important to centralize the role of justification in relation to the mathematics classroom community and teachers’ professional responsibilities.

While focusing on justification as a learning practices is powerful, at the same time, we do not want to marginalize justification’s defining purpose – it verifies. And because of this core feature, justification is central to expanding a disciplinary knowledge base of mathematics. It is a knowledge-producing activity. Over the course of their schooling, students should develop an increasingly sophisticated understanding of how mathematical ideas are generated and established, and increasing confidence that they can do this work themselves. Each discipline has its own methods of establishing knowledge (disciplinary modes of justification) and mathematical justification is a unique process. Understanding mathematical justification requires considering how we know what we know, and determining how we assess the validity of claims based on other known information.

The outcome of understanding mathematical justification as a process of verification and knowledge production connects strongly with a goal of a democratic education as well – to produce a citizenry that can evaluate claims and generate knowledge together by engaging in deliberative discussions with others (Dewey, 1916). Thus, we do not want to value justification only as a learning practice, just as we do not want to value it singularly as a mathematical practice, whether focused on verification (knowledge development, establishing results) or more broadly conceived.

It is critically important that teachers understand that the defining characteristic of a justification is that it demonstrates the truth of a claim, regardless of the particular purpose they attend to in any teaching-learning interaction. The practice of justification is rich and multi-faceted, and justification as a learning practice can serve many of the identified roles only because of its core function of verification. For example, the purpose of promoting conceptual understanding is supported because justification involves verification. As students deduce new information from what is already understood, they are identifying and enriching the relationships and interconnection among the mathematical ideas. (Hanna and Barbeau (2008) discuss this idea in relation to potential learning opportunities of doing proofs in high school mathematics beyond learning the result of the proof itself.) Similarly, the purpose of influencing social relationships is possible because justifications aim to verify. Each student has access to the mathematical tools to support his or her position, and ultimately to decide whether he or she is correct, and whether someone else’s argument is correct.1 This is quite different than a social studies class, where, for some topics, one can agree to disagree.

7.2. Shifting the discourse

These results and proposed rationale might broaden the field of mathematics education discourse about justification in the classroom. Whereas the discourse has been shaped by a focus on the purposes of mathematicians, we hope that this study prompts more careful attention to the nature of the teachers’ work and the purposes of education. The focus on mathematicians’ purposes narrows the terrain we explore, and can lead to a matching process, where teachers either use justification for purposes that align with the mathematician community (generally seen as good), or they do not (generally seen as not good). If we instead focus on the value of the practice from the teachers’ perspectives, as they work to meet their responsibilities as educators – responsibilities shaped by a broader context – we are more likely to find value in the present work teachers are doing and are more likely to be able to build on it. Following Chazan and Lueke (2009), we hope this work “challenges researchers interested in relationships between mathematical activity in schools and in the discipline to understand better why certain kinds of mathematical activity rarely find their way into the institutional setting of school” (p. 37). Understanding the practice of justification within the context of teaching is an important step in this agenda. This kind of shift in framing is productive, moving us towards a stance where the aim is to enrich teaching and understand what is present that is of value while simultaneously considering what of potential value is absent.

7.3. Working with teachers towards change

These results have implications for the organization of professional development activities. By better understanding how teachers value and incorporate justification in their classrooms, we are better positioned to work with teachers to support the development of their practice and help them see justification as valuable and useful. As with all learning, it is important to build on the strengths and commitments of those engaged in the learning process. To the degree that teachers have a focus on the student more holistically, it is important to pursue the documentation of purposes that may not be singularly mathematical. Teachers, who may be less concerned with representing mathematics in a way that adheres to the

1 We acknowledge that the social act of articulating a justification – which requires proficiency with language, a degree of confidence, and an environment that welcomes such contributions – may not be available to all equally, at least initially in a class. The fact that justifications verify means that the potential is there, with appropriate pedagogical support from teachers.
mathematician community, may be more engaged – as a first step – by incorporating justification for its value in formative assessment or developing students’ intellectual independence. The findings from this study offer a starting set of purposes that one group of teachers identified that may support this kind of work. Teacher educators could work with teachers to establish justification as part of the teachers’ repertoires and then expand the valued uses, ultimately to ensure that the verification purpose of justification is salient to the teachers both intellectually and in their classroom practice.

8. Conclusion

In this paper, we explored six purposes of justification in the middle grades mathematics classroom communities articulated by one group of 12 teachers. These teachers used justification to meet their main obligation as teachers (e.g., promoting, monitoring and assessing the mathematics learning of a diverse group of students) as well as pursue other valued outcomes (e.g., developing important skills and dispositions for adulthood). We identified some important places of confluence (e.g., explanation) and divergence (e.g., verification) between the purposes of the mathematician community and the mathematics classroom community.

These results broaden our understanding of the ways justification might “be practiced” in middle grades classrooms, prompting us to further consider the nature of two important, but distinct communities – the mathematician and the mathematics classroom – and how the community’s purpose shapes the practices and meaning of practices within the community. This study demonstrated that, even in classrooms that do support justification as a practice, the purposes for engaging in justification may differ in important ways from those of the mathematician community. The meaning of a practice comes from its use in a community and the value of that practice in that community, which in the case of the mathematics classroom community is educating students.

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Appendix A.

BEFORE THE LESSON

1) a) What is the primary goal of this lesson?
   b) What role will justification play in this lesson?

AFTER TEACHING THE LESSON, watch the lesson:

4) Pick one instance involving student justification that you thought was productive and/or went well. Identify the time on the tape and explain what you valued about this instance.

5) Pick one instance involving student justification that didn’t go as well as it could have, and transcribe that exchange between the student(s) and yourself. (Please include the times on the tape.) What would you do next time with this exact situation to make it go better? Explain why you revised it as you did.

6) b) What, if anything, do you think students learned about justification as a result of this lesson?
   7) What, if anything, did you learn about teaching to promote student justification as a result of this lesson? Please be sure to explain how this lesson led to that learning.

8) Given where your students are with justification right now,
   a) What goals do you have for your students with respect to justification?
   b) What will you do next time to move towards these goals? (When you answer this question, think specifically about your interactions with students.)

9) Anything else on your mind with respect to justification or your students’ work with justification.

References
