Studying the classroom implementation of tasks: High-level mathematical tasks embedded in ‘real-life’ contexts

Andreas J. Stylianides\textsuperscript{a,}\textsuperscript{*},\textsuperscript{1}, Gabriel J. Stylianides\textsuperscript{b,}\textsuperscript{1}

\textsuperscript{a}Department of Education, University of Oxford, 15 Norham Gardens, Oxford OX2 6PY, UK
\textsuperscript{b}University of Pittsburgh, Pittsburgh, USA

Received 6 January 2007; received in revised form 30 October 2007; accepted 7 November 2007

Abstract

Mathematical tasks embedded in real-life contexts have received increased attention by educators, in part due to the considerable levels of student engagement often triggered by their motivational features. Nevertheless, it is often challenging for teachers to implement high-level (i.e., cognitively demanding), real-life tasks in ways that exploit their motivational features without overshadowing the mathematics involved. This paper proposes an analytic framework for describing and explaining the classroom implementation of different kinds of tasks, and uses this framework to analyse a classroom episode where a secondary teacher implemented with low fidelity a high-level, real-life mathematical task. Implications for research are discussed.

© 2008 Elsevier Ltd. All rights reserved.

Keywords: Classroom teaching; Curriculum implementation; Mathematics education; Mathematical reasoning; Tasks; Textbooks

1. Introduction

The mathematical tasks implemented in classrooms influence students’ opportunities to learn mathematics in important ways. Specifically, mathematical tasks convey messages about what mathematics is and what doing mathematics entails (Hiebert & Wearne, 1993; Marx & Walsh, 1988; National Council of Teachers of Mathematics [NCTM], 1991; Watson & Mason, 2005; Zaslavsky, 2005), and, thus, the mathematical tasks used in the classroom can limit or broaden students’ views of the subject matter with which they engage (Henningsen & Stein, 1997; Schoenfeld, 1992). Furthermore, mathematical tasks used in the classroom become the objects of students’ activity, and “the setting of the tasks together with related actions performed by the teacher constitute the major method by which mathematics is expected to be conveyed to the students” (Christiansen & Walther, 1985, p. 244).

Despite the significant role that mathematical tasks can play in students’ opportunities to learn mathematics, the nature of tasks selected or designed by teachers for use in the classroom does not predetermine the quality of instruction associated with the implementation of these tasks. For example, research shows that the implementation of tasks is mediated by teachers’ experience, knowledge, and beliefs (Ben-Peretz, 1990; Collopy, 2003; Corey & Gamoran, 2006; Henningsen & Stein, 1997;
Remillard, 1999, 2005; Stein, Grover, & Henningsen, 1996; Stodolsky, 1989). Research also shows that tasks intended by their designers (e.g., textbook authors, teachers) to engage students in high-level (i.e., cognitively demanding) mathematical activity are frequently implemented in the classroom in ways that reduce their complexity (e.g., Doyle, 1983, 1988; Henningsen & Stein, 1997; Hiebert et al., 2003; Stein et al., 1996). This is problematic because, when reduction in complexity occurs, “the cognitive demands of the task are weakened and students’ cognitive processing, in turn, becomes channeled into more predictable and (often) mechanical forms of thinking” (Henningsen & Stein, 1997, p. 535).

Numerous classroom-based factors have been identified to contribute to the low fidelity of implementation of high-level tasks (see Bennett & Desforges, 1988; Davis & McKnight, 1976; Doyle, 1988; Henningsen & Stein, 1997; Schoenfeld, 1988; Stein et al., 1996), where by fidelity of implementation of tasks we mean the extent to which the tasks are enacted by teachers and their students in the classroom in the ways they were designed to be enacted by textbook authors, teachers themselves, etc. Some of these factors relate to the inappropriate amount of classroom time allotted to the tasks (either too much or too little time), the lacking design features of the tasks (e.g., limited motivational power or unclear expectations), and classroom-management problems (Henningsen & Stein, 1997). Furthermore, the low fidelity of implementation of high-level tasks has been associated with the fact that the work required by these tasks is often slow and not smooth, student involvement and productivity tend to be low, and the rates of student errors and non-completion of work are high (Doyle, 1988). In addition, the increased effort required by students to do well on these tasks often causes students to seek (directly or indirectly) ways to reduce the risk associated with task engagement, thereby contributing to decline in the tasks’ cognitive demands (Davis & McKnight, 1976).

This paper is situated in the body of research that examines the implementation of high-level tasks in mathematics classrooms and the factors associated with the decline in the cognitive demands of these tasks. Our focus is on high-level tasks that are embedded in real-life contexts. We define real-life contexts broadly to include situations that refer (directly or indirectly) to everyday activities or concern mathematical applications in different disciplines such as science, business, engineering, and economics (Stylianides, 2005).

The special category of tasks that are embedded in real-life contexts has received little attention in the existing body of research on high-level tasks, even though several researchers and reform curriculum frameworks in several countries have called for increased emphasis on tasks that promote applications and connections of mathematics to the ‘real world’ (e.g., Boud & Feletti, 1991; NCTM, 1989, 2000; Romberg, 1992; Schiefele & Csikszentmihalyi, 1995; Streefland, 1991; Trafton, Reys, & Wasman, 2001). In part due to these calls, mathematical tasks embedded in real-life contexts are abundant in reform-based mathematics textbooks (Stylianides, 2005; Trafton et al., 2001). Trafton et al. (2001) observed, for example, that in the United States “[t]he use of applications to contextualize mathematical study is an important characteristic of standards-based [i.e., reform-based] materials” (pp. 263–264). One important feature typically attributed to tasks embedded in real-life contexts is high motivational power. As Hiebert et al. (1996) observed, “it is often proposed that the problems with which students will become most easily engaged are those that are taken from their everyday lives” (p. 18). For example, Trafton et al. (2001) noted that well-designed real-life tasks “stimulate student interest and engagement and the development of a healthy, accurate view of mathematics as a useful discipline” (p. 264). Similarly, Schiefele and Csikszentmihalyi (1995) suggested that teachers can place problem solving in real-life contexts as one way to increase their students’ motivation in mathematics. Yet, the worthiness of tasks embedded in real-life contexts is a contentious issue in the current literature. Some researchers expressed the concern that, even though real-life tasks may seem to motivate students, they may not in fact serve mathematics well. For example, Filloy and Sutherland (1996) argued that algebra tasks embedded in real-life contexts can shift teachers and students’ attention from algebraic methods towards solution methods like ‘trial and refinement’ that support a new definition of school algebra as “something which is far removed from algebra as recognised by mathematicians” (p. 154). Furthermore, some researchers questioned the belief that the real-life aspect of tasks is the primary
determinant of how students engage with them. In particular, Hiebert et al. (1996) challenged the view that the source of students' interest and motivation is inherent to the task and advanced the view that the basis for how students engage with a task is students' prior knowledge and the conditions under which the task is completed (e.g., the values and expectations that have been established in a classroom).

In this paper, we are not concerned with the relative worthiness of real-life tasks and of tasks with no outside-of-mathematics contextualisation. Rather, we identify the context of high-level, real-life tasks as a potentially important factor to consider when examining their fidelity of implementation because of the difficulties that presumably are entailed for teachers in exploiting the motivational aspects of such tasks without however losing sight of the mathematics involved. To investigate the role of real-life context in the implementation of high-level tasks, we analyse an episode where an experienced secondary teacher implemented in her classroom a mathematical task that was embedded in a real-life context and was intended by the textbook authors to engage students in high-level mathematical activity. To support our analysis of the episode, we propose and use an analytic framework for studying the implementation of different kinds of tasks (not necessarily high level or mathematical) in classroom settings. Our analysis of the episode shows that the task was implemented in a way that countered its intended goals and led to decline in its cognitive demands. Also, our analysis suggests that this decline resulted from the interaction, during the implementation phase of the task, between main features of the task (namely, its motivational aspects and its high cognitive demands) and the social practices that regulated the functioning of knowledge in the particular classroom. Using our examination of the episode as a case in point, we discuss implications of the analytic framework for research on task implementation. Also, we discuss the challenges surrounding the classroom implementation of high-level, real-life mathematical tasks in ways that exploit their motivational aspects without however overshadowing the underlying mathematics.

2. An analytic framework for studying the implementation of tasks in classroom settings

A useful analytic framework for studying the implementation of tasks in classroom settings should promote at least two primary goals. First, the framework should specify an operational definition for researchers to describe the fidelity of implementation of a task in a classroom. This can support, for example, informed inferences about whether and how the cognitive demands of a high-level task have declined during its implementation. Second, the framework should specify a space within which researchers can seek an explanation for the fidelity of implementation of a task; this explanation should have the potential to synthesise different factors that have been identified in the literature as being associated with the implementation of tasks. The framework we propose herein has two primary components, each corresponding to one of the aforementioned goals.

2.1. First component: an operational definition for describing the fidelity of implementation of a task

The operational definition we propose for describing the fidelity of implementation of a task is based on the detailed decomposition of academic tasks that was proposed by Doyle (1988, p. 169). Specifically, Doyle described four general elements of an academic task:

(a) **product**: a goal state or end result to be achieved (e.g., a solution to a word problem, answers to a set of test questions, numbers in blanks on a worksheet, oral responses in class);

(b) **resources**: a problem space or set of conditions and materials available to accomplish the task (e.g., information in textbooks, notes from lectures, conversations with other students, models of solutions supplied by the teacher);

(c) **operations**: the actions involved in assembling and using resources to generate the product (e.g., remembering answers from previous lessons, copying numbers off a list, applying a rule to select appropriate answers, formulating an original algorithm to solve a problem, applying logical reasoning on the basis of accepted truths to derive valid conclusions); and

(d) **accountability**: the importance of the task in the overall work system of the class (e.g., a warm-up exercise might account as a daily grade, whereas a test might account for 30% of the term grade).

These four elements are interdependent in the sense that changes in one of them can cause changes in the others. For example, relaxation of the set of
conditions of a task (cf. resources) can reduce the cognitive level of the actions required for its completion (cf. operations).

The four elements of tasks described above can support detailed descriptions of a task as presented in the textbook (or in the teacher’s lesson plan) and at any given time during its implementation in a classroom. We propose that, by contrasting a sufficient number of descriptions of a task at different phases, one can examine whether and how the task changed during its implementation. The following set of descriptions of a task should normally suffice for this purpose: {description of the task as presented in the textbook, description of the task as announced by the teacher, description of the task as reflected in the classroom activity (notably, as reflected in the students’ work and in the products accepted by the teacher)}. We propose also that the fidelity of implementation of a task be defined in terms of differences (or lack thereof) among the various descriptions. Accordingly, the fidelity of implementation of a task can be understood as a property that takes values on a continuous scale from low to high, with high fidelity of implementation being associated with no (or minor) differences among the various descriptions.

2.2. Second component: a space within which to seek an explanation for the fidelity of implementation of a task

We propose that an explanation for the fidelity of implementation of a task be sought within the space defined by the interactions between features of the task and the context of its implementation. In regard to the context of implementation, we identify two broad factors that can synthesise several different factors that have been identified in the literature as being associated with the fidelity of implementation of tasks such as the teacher’s experience, knowledge, and beliefs (Ben-Peretz, 1990; Collopy, 2003; Corey & Gamoran, 2006; Henningsen & Stein, 1997; Remillard, 1999, 2005; Stein et al., 1996; Stodolsky, 1989). For example, a teacher with weak content knowledge may not be able to recognise subtle differences in various products generated by students for a task that may be significant from a mathematical point of view and, thus, consider these differences as being unimportant. Also, a teacher with weak content knowledge may not be able to recognise that seemingly unimportant changes in the expected product of a task may have significant implications for other aspects of students’ mathematical activity in the task, especially in regard to the operations required for generating the modified product.

2.2.2. The custom of the classroom where the task is being implemented

Our use of the concept of custom follows that of Balacheff (1999):

Custom is understood... as a set of obligatory practices (Carbonnier, 1971) established as such by their use, and which, in the majority of cases, is established implicitly. Custom regulates the way in which the social group [e.g., a classroom] expects to establish relationships and interactions among its members and, therefore, it is initially characterized as a product of social practices. (p. 25; italics in original)

Balacheff (1999) used classroom data to exemplify the argument that the custom of a classroom offers a useful space for describing and explaining social practices that regulate the functioning of knowledge in the classroom. In particular, the
custom of a classroom can help explain why the didactical contract specific to the implementation of a given task can be negotiated in different ways in different classrooms, whereby didactical contract means the system of reciprocal obligations between a teacher and his/her students that are specific to the target knowledge (Brousseau, 1997). Thus, the custom of a classroom can help explain also variations in different implementations of the same task. According to Balacheff (1999), the concept of custom may circumscribe the domain of validity of the concept of didactical contract, in the sense that the didactical contract can be seen as having a local character specific to the implementation of a particular task whereas the custom can be seen “as regulating the social functioning of a given class across time” (Balacheff, 1999, p. 26). Balacheff (1999) noted further that the “[c]ustom matters at the moment of negotiation of the didactical contract, in particular in determining what is negotiable and what is not. When the contract vanishes, the class comes back to its usual custom.” (p. 26)

Many classroom-based factors that have been identified in the literature as being associated with the fidelity of implementation of tasks can be expressed in terms of the social practices that regulate the functioning of knowledge in a classroom and that comprise the custom of the classroom. For example, two inter-related factors that have been identified for the low fidelity of implementation of high-level tasks are that the work required by these tasks is often slow (leading to low productivity) and not smooth (leading to high rates of student errors) (Doyle, 1988). These factors relate to the social practices in a classroom that regulate the expectations of the teacher and the students regarding how much time might be required for the solution of assigned tasks and how much difficulty might be reasonable for students to encounter when they try to solve the tasks. In a classroom where it is customary for students to be able to solve correctly assigned tasks within a few minutes, it is anticipated that high-level tasks will be implemented with low fidelity (cf. Lefstein, 2005).

3. Method

3.1. Data and context

The data related to the focal classroom episode are derived primarily from an extended set of fieldnotes that the two of us collected during classroom observations over a 4-month period in a seventh-grade class in a suburban school in the United States. We had arranged with the teacher to observe one two-period mathematics lesson every week with the intention to improve our understanding of issues related to the implementation of reform-based mathematics textbooks. Occasionally, the two of us observed the same lessons (this also happened the week of the focal episode), but most of the times we observed different lessons. In all occasions, however, we discussed our observations after each lesson. Also, we sometimes debriefed lessons we observed with the teacher. Our fieldnotes on the focal episode are supplemented by data from a semi-structured interview (Merriam, 1988) with the teacher that we conducted jointly at the end of the 4-month period. The interview was audiotaped and fully transcribed. Our goals for the interview were to elicit the teacher’s thoughts about the episode and, more broadly, her perspective on different issues related to teaching and learning mathematics (such as how students learn mathematics best).

The teacher of the classroom, called Nancy (pseudonym), was experienced and had the reputation of being an exemplar mathematics and science teacher in the school district. Indeed, a few months after the end of our classroom observations, Nancy received the prestigious US Presidential Award for Excellence in Mathematics and Science Teaching. The Presidential Awards are the highest honours for teachers of mathematics and science in the United States, and are bestowed upon “a premier group of highly qualified teachers who have both deep content knowledge of the subjects they teach and the ability to motivate and enable students to be successful in these areas”.3

In regard to our role in the classroom, we did not participate in the teacher’s decisions of what was to be taught and how. During whole class discussions, we were sitting at the back of the room, taking fieldnotes. During small group work, however, we were circulating around, listening to the students’ conversations, asking questions to elicit students’ thinking (e.g., “How did you figure this out?”), paying attention to the scaffolding offered by the

---

teacher to the students, and taking notes on our observations.

The classroom offered a safe environment where the teacher and the students shared a feeling of mutual respect, and the students felt free to share their ideas with the teacher and with one another. The class was organised as a community wherein the students were expected to collaborate in their groups to solve tasks assigned by the teacher. Most of the times, the work of the class on the tasks was completed in the small groups, without a whole class discussion for explanation and negotiation of the intellectual constructions of the small groups. The small groups were accountable to the teacher and their work would officially end once the teacher ratified it. Further information and evidence about the characteristics of the classroom environment are offered in Section 6.2.1.

The tasks in which the students engaged were taken from the mathematics textbook series Nancy was using (described in detail below), and many times the tasks were embedded in a real-life context. Nancy considered students’ engagement with real-life tasks as crucial for their learning. This is indicated in her response to an interview question about how she believed students learn mathematics best:

Interview excerpt #1: I think students learn math best in a setting where they use mathematics to do something they really want to do. So, they have a problem that they want to solve for a reason and mathematics is the tool leading to solve the problem… So, I think that mathematics based on real life and really engaging problems for kids is how kids learn math best.

Nancy’s view of the utility of real-life tasks in students’ learning of mathematics, especially in regard to the presumed power of these tasks to engage students and stimulate their interest, is aligned with the point made in the literature about the motivational power often attributed to this kind of tasks (cf. Hiebert et al., 1996; Schiefele & Csikszentmihalyi, 1995; Trafton et al., 2001). Also, Nancy’s emphasis on motivation as a way of supporting her students’ learning is consistent with one of the criteria for the teaching award she received, namely, the teacher’s ability to motivate students.

The classroom followed the Connected Mathematics Project (CMP) (Lappan, Fey, Fitzgerald, Friel, & Philips, 1998a/2004a), which is the most popular reform-based middle-school (grades 6–8) mathematics textbook series in the Unites States (US Department of Education, 2000). CMP is a complete mathematics textbook series with student and teacher support materials that aim to embody the guidelines set forth by the curriculum reform frameworks Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) and Principles and Standards for School Mathematics (NCTM, 2000).

CMP is organised around problem solving activities and its overarching goal is “to help students and teachers develop mathematical knowledge, understanding, and skill, as well as awareness and appreciation of the rich connections among mathematical strands and between mathematics and other disciplines” (Lappan, Fey, Fitzgerald, Friel, & Philips, 2002, p. 1). In part due to its emphasis on connections, CMP promotes contextualisation of students’ mathematical activity. Textbook analysis of the algebra, geometry, and number theory units of CMP showed that 45% of the tasks in these units were (directly or indirectly) making reference to everyday activities or concerned mathematical applications in other disciplines (Stylianides, 2005). In other words, almost half of the tasks in these units were embedded in real-life contexts.

3.2. Selection of the episode

As we explained earlier, Nancy was using a textbook series that placed high emphasis on real-life tasks and she believed that real-life tasks are the best way to promote student learning in mathematics. The episode we selected for analysis and discussion in this paper is representative of a phenomenon we observed several times in Nancy’s classroom: during the implementation of high-level tasks embedded in real-life contexts, the motivational aspects of these tasks (e.g., drawing pictures of people’s faces with different scale factors) tended to overshadow the underlying mathematics (e.g., the notion of similarity). By studying in detail an instantiation of this phenomenon in Nancy’s classroom, we aim to offer insights into the challenges that even an experienced and distinguished teacher can have to implement high-level, real-life tasks in a way that utilises their motivational aspects without losing focus of the mathematics involved.

3.3. Analysis

The analysis of the focal episode proceeds in two stages. In the first stage, we use the first component
of the analytic framework to describe the fidelity of implementation of the task in the episode, which turns out to be low. In the second stage, we use the second component of the framework to account for the decline in the cognitive demands of the task that occurred during its implementation.

4. The episode

The class period started with a brief discussion of the homework assignment from the previous day. After the discussion of this assignment, Nancy referred the students to their textbooks and introduced the ‘Trip Task’ (see Fig. 1).

Nancy asked a student to read aloud the introductory paragraph of the task together with the first two notes that Malcolm and Sarah wrote about their trip. She drew the students’ attention to the phrases “we rode into a strong wind” (see note 1 [N1]) and “the wind shifted to our backs” (N2), and she asked the students to comment on them. Some students pointed out that the speed with which Malcolm and Sarah travelled would be low in the former case and significantly higher in the latter case. Then, Nancy asked for some approximate values of the distance covered per hour in each of the two cases. She highlighted to the students that they needed to come up with reasonable estimates based on their everyday experiences. A student suggested 5 miles/h when the riders were pedalling against the wind and 9 miles/h when the wind shifted to their backs. Nancy and the other students considered these estimates reasonable. Nancy emphasised that, in their work on the task, the students should take into consideration all six notes: “Each note has something important to tell us”.

Next, Nancy asked the students to work in their groups (four or five students in each group) to first fill in a table that she gave them to facilitate their work on the task (see Fig. 2) and then to make a graph to represent the information in the table. Nancy also explained to the students that the product of the activity would be a poster of the trip that would include the table, the graph, and some drawings for the different stages of the trip. For example, the students could draw some children swimming for N4 or a van for N5. Nancy then showed to the students, for about a minute, a

Day 4: Chincoteague Island to Norfolk:

On day 4, the group traveled from Chincoteague Island to Norfolk, Virginia. Norfolk is a major base for the United States Navy Atlantic Fleet. Malcolm and Sarah rode in the van. They forgot to record the distance traveled each half hour, but they did write some notes about the trip.

Malcolm and Sarah’s Notes

1. We started at 8:30 a.m. and rode into a strong wind until our midmorning break.
2. About midmorning, the wind shifted to our backs.
3. We stopped for lunch at the barbecue stand and rested for about an hour. By this time, we had traveled about halfway to Norfolk.
4. At around 2.00 p.m., we stopped for a brief swim in the ocean.
5. At around 3.30 p.m., we had reached the north end of the Chesapeake Bay Bridge and Tunnel. We stopped for a few minutes to watch the ships passing by. Since bikes are prohibited on the bridge, the riders put their bikes in the van, and we drove across the bridge.
6. We took 7½ hours to complete today’s 80 mile trip.

A. Make a table of (time, distance) data that reasonably fits the information in Malcolm and Sarah’s notes.
B. Sketch a coordinate graph that shows the same information.

Follow up:

Explain how you used each of the six notes to help you make your table and graph.

Fig. 1. The ‘Trip Task’ as presented in the textbook (Lappan et al., 1998b/2004b, pp. 23–24).
sample poster that a group of students produced the previous year. The graph in that poster was similar to the one suggested by the CMP authors in the teacher’s edition (Lappan, Fey, Fitzgerald, Friel, & Philips, 1998b/2004b, p. 35h) and looked like the graph in Fig. 3.

After this introduction to the task, the students started working in their small groups. The students showed a lot of interest in the task and were excited to create their posters. The teacher was circulating around, offering guidance and asking probing questions when she saw a group struggling. A typical question she was asking was: “What is something that you absolutely know from the clues [i.e., Malcolm and Sarah’s notes]?” She also encouraged the students in each group to exchange ideas about how they might use the information in the six notes.

The class period ended before the students were able to complete the task. Most of the groups finished the table and were about to start making the graph. The groups completed the task at the beginning of the next class period. A typical graph made by four out of the six groups looked like the one in Fig. 4. The graphs made by the other two groups looked like the one in Fig. 3. A fundamental difference between the graphs in the two figures is that in Fig. 4 the slopes of the line segments that represent movement (bike riding under different conditions and ride in a van) have (almost) the same magnitude, whereas in Fig. 3 the slopes of the line segments that represent movement have different magnitudes to appropriately represent variations in speed.

The class work on the task ended with the small group work; there was no whole class discussion for the students to explain their products and to reflect on the differences among them. The teacher said to the students that she was very satisfied with their products and she characterised as ‘excellent’ all the posters. The posters were displayed on the board for several days.

5. Describing the fidelity of implementation of the task in the episode

In this part, we use the first component of the analytic framework to describe the fidelity of implementation of the ‘Trip Task’ in the episode. Specifically, we use Doyle’s (1988) four general elements of academic tasks to contrast (1) the task as presented in the textbook and as announced by the teacher, and (2) the task as announced by the teacher and as reflected in the classroom activity (notably, in the students’ work and in the products accepted by the teacher). Our analysis is informed by data from the interview we conducted with Nancy. These data shed light on specific decisions that Nancy made during the implementation of the task.
5.1. **The task as presented in the textbook and as announced by the teacher**

5.1.1. **Product**

The expected product of the task as presented in the textbook was for the students to use the narrative notes in order to make a table and a graph that would reasonably fit the data. When the teacher introduced the task to the class, she asked the students to create a poster of the trip that would not only include the table and the graph but also some drawings that would show the different parts of the trip. As Nancy explained during the interview, her decision to ask the students to make a poster was influenced by a professional development workshop she had attended:

*Interview excerpt #2: The making of a poster idea actually I stole from... a workshop I took in teaching Connected Math [i.e., the CMP curriculum]. I really liked the way that worked.*

Her reasoning for asking the students to include drawings in their posters was that the drawings (1) could replace the ‘follow-up’ part of the task (cf. Fig. 1), which was asking students to explain how they used each of the six notes to make their tables and graphs, and (2) would inject fun into students’ engagement with the task:

*Interview excerpt #3: I wanted them to add some kind of drawings to the graph, showing what notes they used. Because in the book it asks them to explain how they used each of the clues, each of Malcolm’s and Sarah’s notes to make their graph and table... Some of the kids... they really liked the drawings on the sample [poster] I held up... The pictures come out from the kids, because they like that creative idea with the poster... The poster I did brings up different kinds of strength from different kids, and it also brings them working together with a really fun way for a product of the team... So, I like to inject fun and that was a lot of fun... [She is laughing,]... It was stressful but it was fun! They struggled with that, didn’t they?*

5.1.2. **Resources**

Nancy made two significant additions to the set of materials available for completing the task as presented in the textbook. First, she provided the students with a table (cf. Fig. 2) that included not only the independent and dependent variables (time and total distance, respectively) but also a specific scale for the independent variable (half-hour scale). Nancy explained during the interview that she provided the table to save time from having the students create their own tables:

*Interview excerpt #4: The reason I provided the form for the table was that in earlier years it took so much time with the ruler, you know, doing a table with the ruler... It becomes so much of a production that I have to decide how do I want the time to be spent. And so I decided in this case that I don’t want the time be spent drawing lines.*

Second, Nancy presented briefly to the students a good example of the expected product of the task, namely, a poster that some of her students created the previous year. As Nancy explained during the interview, her students often needed more structure than that offered in the textbook in order to be clear on what was expected of them in some tasks:

*Interview excerpt #5: Sometimes I provide a little more structure. I found that the questions in the book sometimes, when you read them, even if...*
you read them very carefully, it is not clear what they are asking for. The students might know the mathematics but... is hard to figure out what the question is asking for. So I might provide a table or, you know, a form, or making sure... or more questions: have you answered this graph, have you answered this equation?

Nancy did not make any major changes to the set of conditions for completing the task. Specifically, she emphasized the point mentioned in the textbook that the students were expected to make a table and a graph that would reasonably address all constraints described in Malcolm and Sarah’s notes.

5.1.3. Operations

The task as presented in the textbook required five main operations for its completion:

1. Interpreting the real-life activities described in Malcolm and Sarah’s notes to draw reasonable inferences about the distance covered in certain time intervals.
2. Identifying a convenient scale for the time variable of the trip (a half-hour scale would be a strategic choice because all the notes in the task described activities that lasted multiples of half an hour or began/ended at full/half hours).4
3. Making a table of (time, distance) data based on the inferences drawn in operation #1 and using the time scale identified in operation #2.
4. Sketching a coordinate graph that shows the information in the table.
5. Explaining how each of Malcolm and Sarah’s notes were used to make the table and sketch the graph.

These five operations describe a high-level mathematical task in the context of a real-life situation. Specifically, the operations require the following high-level cognitive processes for their completion: drawing inferences about different values of quantitative variables based on information from a real-life situation (cf. operation #1), making strategic choices to represent numerical data in meaningful ways (cf. operation #2), representing numerical data in tabular form (cf. operation #3), representing data from one form to another (cf. operation #4), and explaining one’s reasoning (cf. operation #5).

As we explained earlier, when Nancy announced the task to the class she made some changes to the expected product and the set of resources available for completion of the task. Next we discuss how these changes affected the five operations required for the completion of the task.

Motivated by the need to inject some fun into the activity, Nancy replaced the ‘follow-up’ part of the task, which was asking students to explain how they used each of the six notes to make their tables and graphs, with the requirement that the students would “add some kind of drawings to the graph [that would show] what notes they used” (cf. interview excerpt #3). This change in the expected product replaced operation #5 with operation #5′:

5′. Making drawings to show what notes were used in each part of the coordinate graph.

Nevertheless, the operation of showing what notes were used required significantly lower cognitive activity than the operation of explaining how each note was used. An appropriate explanation for how each note was used would not only indicate what note was used in each part of the graph, but would specify also why that way of using the note was sensible. Had the requirement for drawings been in addition to the requirement for explanation, this would serve Nancy’s goal to inject fun into the activity and would preserve the level of cognitive demands of the task.

We turn now to the two significant changes that Nancy made to the set of resources available to students for completing the task. First, she provided the students with a table (cf. Fig. 2), which included, in addition to the independent and dependent variables, a specific scale for the independent variable (time). By so doing, Nancy replaced operation #2 with operation #2’ , which required lower cognitive activity than the original operation:

2′. Using a given scale for the time variable of the trip.

Had Nancy given to her students a table with no scale, this would have saved students the time needed to draw their own tables (as Nancy intended) and, at the same time, would have not reduced the cognitive demands of the task.

Second, Nancy presented to the students, for about a minute, a sample of the expected product of the task, namely, a poster that some of her students created the previous year. It is reasonable to hypothesize that students’ exposure to the sample

---

4The sentence in the introductory paragraph of the ‘Trip Task’, which says that Malcolm and Sarah “forgot to record the distance traveled each half hour”, provides a hint to the students that a half-hour scale would be a strategic choice.
poster motivated them to engage with the task and want to create their own posters. Nancy noted during the interview: “Some of the kids … they really liked the drawings on the sample I held up… The pictures come out from the kids, because they like that creative idea with the poster” (cf. interview excerpt #3). One could argue, though, that the sample poster facilitated students’ work related to operation #1, because it offered the students a sense of the general form of the function in the graph (a non-decreasing function with interchanging intervals of positive and zero derivatives). Nevertheless, given that the students saw the sample poster for a short period of time and given also the complexity of the task, we do not think that, overall, the teacher’s action of showing to the students the sample poster had a significant influence on the cognitive demands of the task.

To conclude, the changes that Nancy introduced to the task in terms of both the expected product and the set of resources available to students for completion of the task made the operations required for successful completion of the announced task to be, on average, of lower cognitive demand than those required for successful completion of the task as presented in the textbook. Nevertheless, the task as announced to the students was still a high-level task, because students had to apply high-level cognitive competencies to complete it: drawing inferences about different values of quantitative variables based on information from a real-life situation (cf. operation #1), representing numerical data in tabular form (cf. operation #3), and representing data from one form to another (cf. operation #4).

5.1.4. Accountability

The task as announced by the teacher to the students had a significant role in the work system of the class. Like it happened with most other high-level tasks solved in the class, the students were expected to work on the ‘Trip Task’ in their small groups according to the guidelines provided by the teacher who would then circulate around offering support as needed. The small groups were accountable to complete their work without counting on a subsequent whole class discussion, because most likely such a discussion would not occur. The work in the small groups would normally conclude once the teacher ratified the students’ products at the group level.

5.2. The task as announced by the teacher and as reflected in the classroom activity

5.2.1. Product

Based on how Nancy announced the task, especially her emphasis on students having to consider carefully and use appropriately all six notes to make their tables and graphs, the students were expected to produce posters with graphs similar to that in Fig. 3. Yet, only two groups produced such graphs. The other four groups produced graphs similar to that in Fig. 4. Furthermore, all six groups drew pictures to indicate what notes they used in each part of the graph, without however providing a mathematical explanation for how they used each note. The teacher’s decision to replace the ‘follow-up’ part of the task with the requirement for drawings was not making it necessary for the groups to explain how they used each note. How each group used the notes remained unknown to classroom members outside the group, because there was no whole class discussion after the groups finished their posters. Nancy ratified all posters and characterised them as ‘excellent’. She made no distinction among the products of the different groups, even though the expectations she expressed when she announced the task were compatible with the products of only two of the groups.

5.2.2. Resources

All groups used the table provided by Nancy (cf. Fig. 2) to record their inferences about the distance covered in half-hour intervals. Regarding the set of conditions for completing the task, the groups that produced graphs like that in Fig. 4 seemed to have interpreted the notes solely in terms of ‘movement’ versus ‘no movement’, representing movement with line segments of approximately equal slope irrespectively of the particularities of the movement (bike ride under different conditions and ride in a van). Thus, we can infer that the work of these four groups was inconsistent with the set of conditions for completing the task as announced by the teacher.

5.2.3. Operations

The two groups that produced graphs similar to that in Fig. 3 seem to have used the following set of operations required for completion of the task as announced by the teacher: {#1, #2, #3, #4, #5} (cf. Section 5.1.3 for elaboration on the elements of this
set). However, the other four groups that produced graphs similar to that in Fig. 4 seem to have replaced operation #1 with operation #1".

1". Interpreting the real-life activities described in Malcolm and Sarah’s notes to draw rough (as opposed to reasonable) inferences about the distance covered in certain time intervals.

The ‘rough’ inferences are reflected in the way these four groups interpreted the notes solely in terms of ‘movement’ versus ‘no movement’. Operation #1" required lower cognitive activity than operation #1 and, therefore, the use of operation #1" signified decline in the cognitive demands of the task. Also, the use of operation #1" affected the products of all other operations (notably, the table and the graph produced by operations #3 and #4, respectively), thereby resulting in a different overall product (i.e., poster).

5.2.4. Accountability

The teacher changed the accountability before the students. There was a gap between the task as announced by the teacher and the task as reflected in the products accepted by the teacher: initially, the task was presented as requiring careful consideration and appropriate use of all six notes, whereas, at the end, these requirements were essentially ignored when the teacher was evaluating the posters produced by the groups. Thus, the implementation of the task in the classroom exemplifies what Henningsen and Stein (1997) referred to as “lack of accountability for high-level products or processes” (p. 537).

6. Explaining the low fidelity of implementation of the task in the episode

To provide a possible explanation for the low fidelity of implementation of the task in the episode we consider, in turn, the two factors specified in the second component of the analytic framework.

6.1. Nancy’s awareness of possible changes in the task at different phases of its implementation in the classroom

As we discussed in Section 5, Nancy made some changes in both the expected product of the task (notably, she asked students to create a poster and add drawings to it instead of answering the ‘follow-up’ question) and the set of resources available to students for completion of the task (notably, she provided students with a table to fill in the values for the dependent variable using a specified scale for the independent variable). Nancy made these changes consciously as part of her efforts to make the activity more enjoyable to students and to save time from having students to create their own tables. Nevertheless, Nancy did not seem to realise that these changes in the expected product and the available resources made the operations required for successful completion of the task to be, on average, of lower cognitive demand than those required for successful completion of the task as presented in the textbook. This observation, however, does not suffice to provide an adequate explanation for the low fidelity of implementation of the task in the episode, because the task as announced by Nancy to the students was still high level: its successful completion required application of high-level cognitive competencies such as drawing inferences about different values of quantitative variables based on information from a real-life situation, representing numerical data in tabular form, and representing data from one form to another.

A critical issue to consider in seeking a possible explanation for the low fidelity of implementation of the task in the episode is whether Nancy was aware of the fact that the products of four out of the six groups that she accepted did not meet the standards that she set when she announced the task. There are three indications that Nancy was aware of these limitations. First, based on our interactions with her over a 4-month period, we observed that she had strong content knowledge. This observation is corroborated by the fact that one of the selection criteria for the Presidential Award she received was that she had strong content knowledge (cf. Section 3.1). Second, when she introduced the task to the students she emphasised to them that they should use all the notes in the task to make reasonable estimates. To make this expectation clear to the students she helped them come up with estimates that were using appropriately the information offered by each of the first two notes in the task (“we rode into a strong wind” [N1] vs. “the wind shifted to our backs” [N2]). This indicates that Nancy was aware of the fact that the slopes of the segments of the graph that corresponded to different notes could not all have the same magnitude (in the products of the four groups the slopes had almost the same magnitude). Third, the
sample poster that Nancy showed to her students when she announced the task did not suffer any of the limitations of the posters produced by the four groups.

If we accept that Nancy was indeed aware of the limitations of the products of the four groups, then we are confronted with the following question: Why did she characterise ‘excellent’ the products of the four groups? We argue that the custom of Nancy’s classroom provides a useful context within which to seek an answer to this question.

6.2. The custom of Nancy’s classroom

6.2.1. General description of the custom of the classroom

Any description of the custom of a classroom is presumptive, because “one only realizes the existence [of custom] when it produces its effects” (Levy-Bruhl, 1964, p. 44; quoted in Balacheff, 1999, p. 25). Our description of the custom of Nancy’s classroom is based on our weekly observations in her class over a 4-month period and on data from our interview with her. Some points in this section are elaborations on our general description of the environment of the focal classroom in Section 3.1.

In the focal classroom, the teacher and the students shared a feeling of mutual respect, and the students felt free to share their ideas with the teacher and with one another. Creating a safe classroom environment was a high priority for Nancy, as this is indicated in the following interview excerpt:

Interview excerpt #6: [I]t’s really important that the students who live in that class feel that it is a safe place for them to think and make mistakes and… that all can feel that safety and that they can offer their ideas out not worrying whether they are wrong or not. That’s really hard because kids already come to us worrying whether they are right or wrong or they are unwilling to share.

Nancy’s belief about the importance of creating a safe classroom environment for her students evolved over the years:

Interview excerpt #7: I remember as a new teacher, you know, looking at papers and wanting to write on them, “Why did you do this? Why did you give me this garbage?” [She is laughing.] This is terrible and I realise, especially when I sit and work with students who came for help or something, they are really struggling. I say, “Look at how kids work on these things and they put their hearts on it”. But when you write things like, “What did you think you were doing?” or “Didn’t you listen to the instructions?” they get hurt. I mean it is really painful because they really tried.

The custom of Nancy’s classroom was such that students’ efforts were highly appreciated and the teacher avoided making comments on the students’ work that could disappoint them. Critical comments on students’ work were considered inappropriate, irrespectively of the quality of students’ work, because such comments could hurt students’ feelings. During each class period, the teacher frequently characterised the students’ responses to her questions as ‘excellent’ or ‘great’, hardly ever distinguishing between more and less mathematically appropriate responses.

Furthermore, Nancy’s classroom was organised as a community wherein the students were expected to work in small groups to solve the tasks announced by the teacher. The small groups were accountable to the teacher and their work would officially end once the teacher ratified it. The students were often motivated to engage in these tasks, presumably due to the good relationships they had with one another and their interest in the tasks. As we noted earlier, Nancy was using a textbook series that placed high emphasis on real-life tasks, which are considered to have high motivational power. Also, Nancy highly valued real-life tasks, as she believed that these tasks are the best way to promote student learning in mathematics (cf. interview excerpt #1).

Nancy placed a lot of emphasis on communicating clearly to her students her expectations when she announced a task (cf. interview excerpt #5). When sometimes her students failed to meet her expectations, she attributed the failure to her poor articulation of these expectations. The latter is reflected in Nancy’s response to an interview question about what she would do if she assigned a task and the student products she received did not meet her expectations:

Interview excerpt #8: I would reflect on what information I gave them and what I asked them for. Because my first assumption is that I failed to communicate effectively with my students. I usually find that if I communicate clearly, and the expectations are clear, my students usually
want to complete the task. We have great kids, I mean, they are motivated and they are trying to do whatever is expected of them. But, usually, when I find a pretty big failure and I don’t get what I thought I was going to get, it’s because I didn’t communicate very well. So, what do I do at that point? If it was a really important lesson, or a really important product they needed to come up with, I would put us back together and say: “Men, I blew it. Let’s look at what you did. Let’s look what you did and what I told you and you guys tell me what your understanding was and how can I make my instructions better”. 

Finally, the work of the class on the assigned tasks was most of the times completed in the small groups, without a whole class discussion for explanation and negotiation of the intellectual constructions of the small groups. This practice often had two interrelated implications for the functioning of knowledge in Nancy’s classroom. The first implication was that there was often lack of homogeneity (Balacheff, 1990, p. 259) in the intellectual constructions of the different groups. In other words, the different groups often produced different intellectual constructions, without the community negotiating the meanings of these constructions to decide whether and how they related with one another (e.g., whether they were mathematically equivalent) and achieve coherence. The second implication was the ambiguity about which intellectual constructions from those that resulted from the students’ work in the small groups actually reached the status of knowledge that could be (1) shared among them and the other members of the classroom community, and (b) retained for future classroom work (for discussions of the practices of ‘sharing’ and ‘retaining’ knowledge in the social functioning of classroom communities, see Balacheff, 1990; Ball & Bass, 2000; Brousseau, 1997; Simon & Blume, 1996; Stylianides, 2007; Yackel & Cobb, 1996).

6.2.2. Using the custom of the classroom to offer a possible explanation for the low fidelity of implementation of the task in the episode

The custom of Nancy’s classroom had four primary characteristics: (1) the teacher highly appreciated students’ efforts and avoided critical comments on their work, trying to instill in them a feeling of accomplishment and to maintain a positive environment in her class; (2) student motivation was considered a vehicle to learning; (3) possible student failure in completing the assigned tasks would be attributed primarily to the teacher’s poor articulation of her expectations; and (4) assigned tasks were most of the times completed in the small groups, without whole class discussion for sharing and analysis of the intellectual constructions of the small groups. The motivational features and the high cognitive demands of the ‘Trip Task’ seemed to have interacted, during the implementation phase of the task, with these four characteristics of the custom of Nancy’s classroom in the following way.

The motivational features of the task (attributed presumably to its real-life context) offered a good match with the second characteristic of the custom above. The students’ interest in the task provoked high levels of engagement with the task that resulted in the generation of aesthetically appealing posters, an indication of the students’ hard work. Thus, the first characteristic of the custom provides an explanation for why the teacher characterised the products of all groups as ‘excellent’. However, the high cognitive demands of the task provoked mathematical activity of various levels in the different groups; this activity supported, in turn, the production of posters of uneven mathematical value. Discussion of these posters in the whole class, including critical examination of the different ways in which some of the groups represented parts of the trip, would most likely reveal the limitations of some of the posters. Revelation of these limitations would most likely create a problematic situation in the classroom, for it could ‘undermine’ the work of the groups that produced these posters (cf. first characteristic of the custom) and/or it could suggest that the teacher failed to articulate clearly her expectations when presenting the task to the students (cf. third characteristic of the custom). The fourth characteristic of the custom ‘saved’ the situation, as it allowed the activity to end without whole class discussion and without the teacher having to assume responsibility for the limitations of the products generated by some groups.

7. Conclusion and directions for future research

Based on our analysis of the episode from Nancy’s classroom and the research finding that classroom work associated with high-level tasks is often slow, not smooth, and with low student involvement and productivity (e.g., Doyle, 1988),
we conjecture that the increased levels of student engagement often triggered by the motivational features of high-level, real-life tasks can support an illusory feeling of accomplishment that can offer way to less-productive patterns of mathematical work. Orchestrating classroom activity that exploits the motivational features of high-level, real-life tasks without however overshadowing the mathematics involved can be challenging even for distinguished teachers like Nancy. Teachers with weak content knowledge, or teachers who are using textbooks that do not support them to understand the mathematical goals of tasks and to appreciate the different levels of mathematical appropriateness associated with possible student solutions, can face additional challenges when they implement high-level, real-life tasks in their classrooms. However, as our analysis of the episode suggests, strong mathematical knowledge is not sufficient for high fidelity of implementation of such tasks: Nancy had the necessary mathematical knowledge to recognise the mathematical limitations of some of her students’ products, but did not seem to realise that these limitations reflected operations of lower cognitive demands than the demands of the operations required for generating the expected product.

Thus, teacher preparation and professional development programmes have a critical role to play in equipping teachers with the necessary mathematical and pedagogical knowledge that will allow them to understand and appreciate not only the mathematical affordances of tasks but also (1) the correspondence between these affordances and specific operations required by students to complete the tasks and (2) the idea that the level of cognitive demands associated with these operations is consequential for students’ opportunities to learn mathematics. The latter is not to say that only operations that require high cognitive demands are important for students’ learning. Indeed, mathematical proficiency is a multi-dimensional construct (see, e.g., Kilpatrick, Swafford, & Findell, 2001) that encompasses a wide range of competencies, some of which require engagement with operations typically associated with low cognitive demands (e.g., application of procedures). Our point, though, is that operations typically associated with high cognitive demands are necessary for deep learning in mathematics and, as such, should not be underemphasised in classroom instruction.

Our analysis of the episode from Nancy’s classroom suggested that the decline in the task’s cognitive demands during its implementation resulted from the interaction between main aspects of the task (notably, its motivational features and its high cognitive demands) and the custom of the classroom (notably, the social practices that regulated the functioning of knowledge in the classroom). This finding is an important contribution to research on the fidelity of implementation of mathematical tasks in classrooms, because factors that have been identified in the literature as being associated with the decline in the cognitive demands of high-level mathematical tasks (such as the amount of time allotted to the task, the design features of the task, classroom management, and knowledge of the teacher) (see Bennett & Desforges, 1988; Davis & McKnight, 1976; Doyle, 1988; Henningsen & Stein, 1997; Schoenfeld, 1988; Stein et al., 1996) could not explain the low fidelity of implementation of the task in the episode.

Further research is needed to examine the implementation of high-level, real-life mathematical tasks in classrooms that have different customs (e.g., classrooms in which the teachers consider appropriate to provide critical, albeit sensitive, feedback to students), controlling for important teacher characteristics (e.g., the level of teachers’ knowledge). This research can support better understanding of how the motivational features of real-life tasks may interact with different features of the custom of a classroom, influencing the fidelity of implementation of high-level tasks. Another issue for future research is how different features of the custom of a classroom may affect the implementation of high-level tasks with no outside-of-mathematics contextualisation. For example, are there any features which when present in the custom of a classroom make it likely that high-level tasks will be implemented with low fidelity, irrespectively of the context in which they are embedded? Finally, another direction for future research is to examine the role of the teacher and of classroom characteristics in situations where low-level mathematical tasks are modified during their classroom implementation in ways that increase their cognitive demands. This kind of modification of a task would still be associated with low fidelity of implementation, but not in the sense that we and other researchers examined low fidelity of task implementation. The analytic framework we proposed in this paper, which is specific neither to high-level tasks nor to mathematical tasks, can support the investigations outlined in this paragraph as well as
similar investigations on the fidelity of the classroom implementation of tasks in other subject areas.

References


Christiansen, A. G., Howson, & M. Otte (Eds.), *Handbook of Mathematics Education* (pp. 193–224). New York: Macmillan.


